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PARAMETRIC INVESTIGATION OF RADOME ANALYSIS METHODS:

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COMPUTER–AIDED RADOME ANALYSIS USING GEOMETRICAL OPTICS AND LORENTZ RECIPROCITY

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By

G. K. Huddleston, H. L. Bassett, & J. M. Newton

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GRANT AFOSR–77–3469
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A Fortran computer program is described for computing the effects of a tangent ogive radome on the receiving patterns and boresight directions of a monopulse antenna. The analytical method based on ray tracing, main program, and 34 subroutines are thoroughly documented. Four test cases and their results in the form of printouts, pattern plots, and near-field graphs are presented.

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Chapter 1

INTRODUCTION AND SUMMARY

1-1. Introduction

This Volume II of this final technical report of four volumes documents a ray tracing radome analysis computer program written in Fortran IV for use on the Cyber 70/74 computing system at Georgia Institute of Technology and the IBM 3033 computing system at Johns Hopkins University Applied Physics Laboratory. The program was developed at Georgia Institute of Technology over the past four years; however, considerable development work in computer aided radome analysis has taken place here prior to that time [1-7].

This analysis package was used during the research carried out under this grant to analyze the antennas and radomes using the fast receiving formulation as described in Volume I. Its documentation was done in conjunction with the on-going radome technology program at JHU/APL under the cognizance of R. C. Mallalieu (APL Contract 601053). It is intended to serve as part of a technology base for the radome technical community.

The report is organized by chapters according to the approximate order in which the subprograms are called, and each chapter describes one subprogram. Each chapter is essentially self-contained since it is meant to serve as the complete documentation on a single subroutine. References are provided at the end of each chapter. In some cases, figures are duplicated in different chapters for completeness. Each chapter is terminated with the listing of the subroutine.

Chapter 2 describes the main program and instructions for its use. Chapters 3 through 28 describe the thirty four subroutines required for execution, including those for producing Calcomp pattern plots and three-dimensional plots. Appendices A through D present computed results for four test cases for use in verifying correct operation on other systems. These results were obtained on the Cyber 70/74 computing system at Georgia Tech. The remaining part of this chapter describes background of the program development and summarizes the features of the computer analysis.

This report comprises Volume II of four volumes. Volume I describes the salient results of this overall investigation to determine the accuracies and ranges of validity of various analysis methods. Volume III documents the additional software required to analyze radomes using a surface integration method. Volume IV presents the experimental results obtained and is meant to serve as a data base for other investigators seeking to verify the accuracy of their computer codes.

1-2. Background

Development of the radome analysis computer program (RACP) was initiated in 1971 in an effort to include the effects of the radome on a ground mapping radar [1]. A three-dimension geometry and vector field formulation were used. A plane wave spectrum (PWS) representation of the radiation from the antenna greatly facilitated the computations since the Fast Fourier Transform (FFT) could be used. The program was used to compute power patterns on the ground for many different cases of antenna/missile orientations. From these data, the effects of the radome on pattern shape, power loss and VSWR were determined.

Monopulse tracking antennas were next introduced into the computer analysis to evaluate radome materials and shapes for seeker systems in the 8-18 GHz band [2]. Tangent ogive shapes of various fineness ratios were analyzed. Monolithic and multilayer wall structures were used. Algorithms were developed to compute boresight errors from the sampled data difference patterns in two orthogonal planes. A modification of this program was also used to conduct a trade-off and development study for the Multipurpose Missile (MPM), later known as ASALM [3].

The next step in the development of RACP came in 1977 with the introduction of a conical scan tracking antenna into the analysis [4]. This antenna necessitated a reformulation of the analysis from the transmitting formulation used earlier to a receiving formulation. The big advantage offered by the latter is that the antenna response can be calculated for only one direction of arrival of the target return (plane wave). In the former, the FFT automatically computes "responses" for many directions of arrival and, hence, is computationally slower. Subsequent versions of the program have used the same receiving formulation with monopulse and other types of antenna models.

The computed results obtained with the receiving and transmitting formulations are not always the same [5]. A computed-aided analysis which utilizes the Huygens-Fresnel principle [6, 7] is generally considered to be more accurate than the two methods already mentioned, but requires considerably more computation time that may not be warranted in all cases. A research program is now underway at Georgia Tech whose objective is to establish the accuracies and ranges of validity of these three methods of radome analysis [5].

1-3. Description of the Analysis

The current version of the ray tracing analysis computer program utilizes a receiving formulation based on the Lorentz reciprocity theorem [5]. A plane wave of selectable linear or circular polarization is assumed incident on the outside of the radome and is represented by a system of parallel rays. There is one ray for each sample data point in the antenna aperture inside the radome. Each ray is traced from the point where it impinges on the outside surface to the corresponding aperture point. The electric and magnetic fields $\underline{E}_i$, $\underline{H}_i$ associated with each ray are weighted by the flat panel transmission coefficients $T_\perp$, $T_\parallel$ as determined by the unit normal $\hat{n}$, the direction of propagation $\hat{k}$, and the dielectric properties of the radome wall. The weighted incident fields $\underline{E}_i'$, $\underline{H}_i'$ at each aperture point are then used in the following integral to obtain the complex voltage response V_r of the antenna as

$$V_r = C \iint_S (\underline{E}_T \times \underline{H}_i' - \underline{E}_i' \times \underline{H}_T) \cdot \hat{z}\, dxdy \qquad (1)$$

where $\underline{E}_T$, $\underline{H}_T$ are the aperture fields when the antenna is transmitting, C is a complex constant, and $\hat{z}$ is the unit vector normal to the xy (aperture) plane. For digital computer implementation, the integral in Equation (1) reduces to a double summation, and the equal-area elements dxdy become $\Delta x \Delta y$ and can be absorbed into the constant C.

In its present form, the program accommodates only one radome shape; viz., the tangent ogive. The length, diameter and fineness ratio are, of course, all variable in the input data. Monolithic and multi-layer wall configurations can be analyzed; however, only uniform wall configurations whose properties do not vary from point to point on the

wall can be handled. Provisions are made to allow for a metal tip on the radome whose effect is aperture blockage.

The geometry subroutines provide for three separate coordinate systems and the point and vector transformations among them. A reference coordinate system is provided to orient the antenna/radome combination with respect to other bodies. The coordinate systems for the antenna and the radome comprise the other two systems. Boresight error and pattern computations are carried out and expressed in the antenna coordinate system.

The primary outputs of the program are boresight error (mrad.), boresight error slope (deg./deg.), gain loss, and when selected, principal plane patterns. Outputs include both printing and plotting (Calcomp). Plotting options allow for selection of aperture fields with and without the radome. A feature is also provided to either obtain or suppress intermediate calculated results for debugging purposes.

Boresight error calculations for monopulse antennas are carried out by setting the first target return at a known direction within a few degrees of true boresight. The responses in the two difference channels and the sum channel are then computed and stored. Another set of responses for a return 180° away from the first is computed next. The two sets of data are then used to construct a linear tracking model in the two orthogonal planes, and the process is repeated until a boresight null is indicated. The true direction of arrival of the plane wave at this point represents the boresight error directly.

The current subroutine used to characterize the antenna permits selection of various polarizations and two aperture distributions. A uniform, circular aperture distribution having vertical, horizontal or

circular (LHR or RHC) polarizations is one combination. The second distribution is a tapered rectangular distribution having vertical polarization as found in flat plate antennas. This basic subroutine would not be difficult to modify to accommodate other distributions, such as rectangular aperture with cosine taper.

Computation time is independent of radome size but depends on the number of samples used in the aperture. For 256 sample points (16 X 16 array), the time to compute the received voltages in the three channels is 1.5 seconds.

The program is organized as a main program and a number of supporting subroutines, all written in Fortran IV. The complete program, including plotting software, contains thirty four subroutines. The core storage required for the complete program, including all library and system I/O routines, is just over 46,000 (decimal) words. Integer, real and complex variables and arrays are utilized. Single, double and three-dimensional data arrays are present. Only single precision variables and computations are required with the 60-bit word available on the Cyber 70 at Georgia Tech.

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10. H. L. Bassett, J. M. Newton, W. Adams, J. S. Ussailis, M. J. Hadsell, and G. K. Huddleston, "Parametric Investigation of Radome Analysis Methods: Experimental Results", Final Technical Report, Volume IV of IV, Grant AFOSR-77-3469, February 1981.

Chapter 2

PROGRAM RTFRACP

2-1. Purpose: RTFRACP is a Fortran computer program used to analyze the effects of a tangent ogive radome on the performance of a monopulse aperture antenna. It consists of a main program and 34 subroutines. It uses complex arithmetic and requires 57121 octal words of core memory for execution on the CDC Cyber 70 system (60 bit words) at Georgia Institute of Technology. Execution time to compute boresight error on the Cyber 70 is approximately two seconds per look direction when the antenna aperture is represented by 16 x 16 = 256 sample data points. Execution time to compute transmitting and receiving patterns and aperture near fields, and to compute the necessary Calcomp commands for two- and three-dimensional plotting, is approximately 35 seconds for one look direction.

The computer-aided radome analysis uses a receiving formulation based on the Lorentz reciprocity theorem as described earlier [1,2]. The voltage produced at the terminals of a linear antenna by an incident plane wave is given by

$$V_R(\hat{k}) = \oiint (\underline{E}_T \times \underline{H}_R - \underline{E}_R \times \underline{H}_T) \cdot \hat{n} \, da \qquad (1)$$

where $\underline{E}_T$, $\underline{H}_T$ are the fields produced on the surface S enclosing the antenna when the antenna is transmitting; $\underline{E}_R$, $\underline{H}_R$ are the incident fields produced on S by the incident plane wave or perturbations thereof; $\hat{k}$ is a unit vector which points from the antenna toward the direction from which the plane wave arrives; and $\hat{n}$ is a unit vector normal to the surface S and pointing

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outward. The fields $\underline{E}_T$, $\underline{H}_T$ are taken to be those produced in the planar aperture when the antenna is transmitting in the absence of the radome. The geometrical optics approximation

$$\underline{H}_T = \frac{\hat{n} \times \underline{E}_T}{\eta} \tag{2}$$

is used to generate the magnetic field in the aperture from the aperture illumination specified by $\underline{E}_T$. Rays are traced from each sample point in the aperture in the direction $\hat{k}$ to the inner radome wall. The plane wave fields associated with each ray are weighted with the flat panel insertion voltage transmission coefficients as determined by the radome wall configuration, the angle of incidence, and the plane of incidence. The individual contributions are summed up as indicated in Equation (1).

The parameters of the tangent ogive radome are indicated in Figure 2-1. The outside base diameter D_{os} and fineness ratio F_{os} determine the outside length according to

$$F_{os} = L_{os}/D_{os} \tag{3}$$

A similar relation holds for the inside dimensions; viz.,

$$F_{is} = L_{is}/D_{is} \tag{4}$$

The radius of curvature of the outside wall R_{os} is given by

$$R_{os} = F_{os} D_{os} / \sin (\pi - 2 \operatorname{Tan}^{-1}(2F_{os})) \tag{5}$$

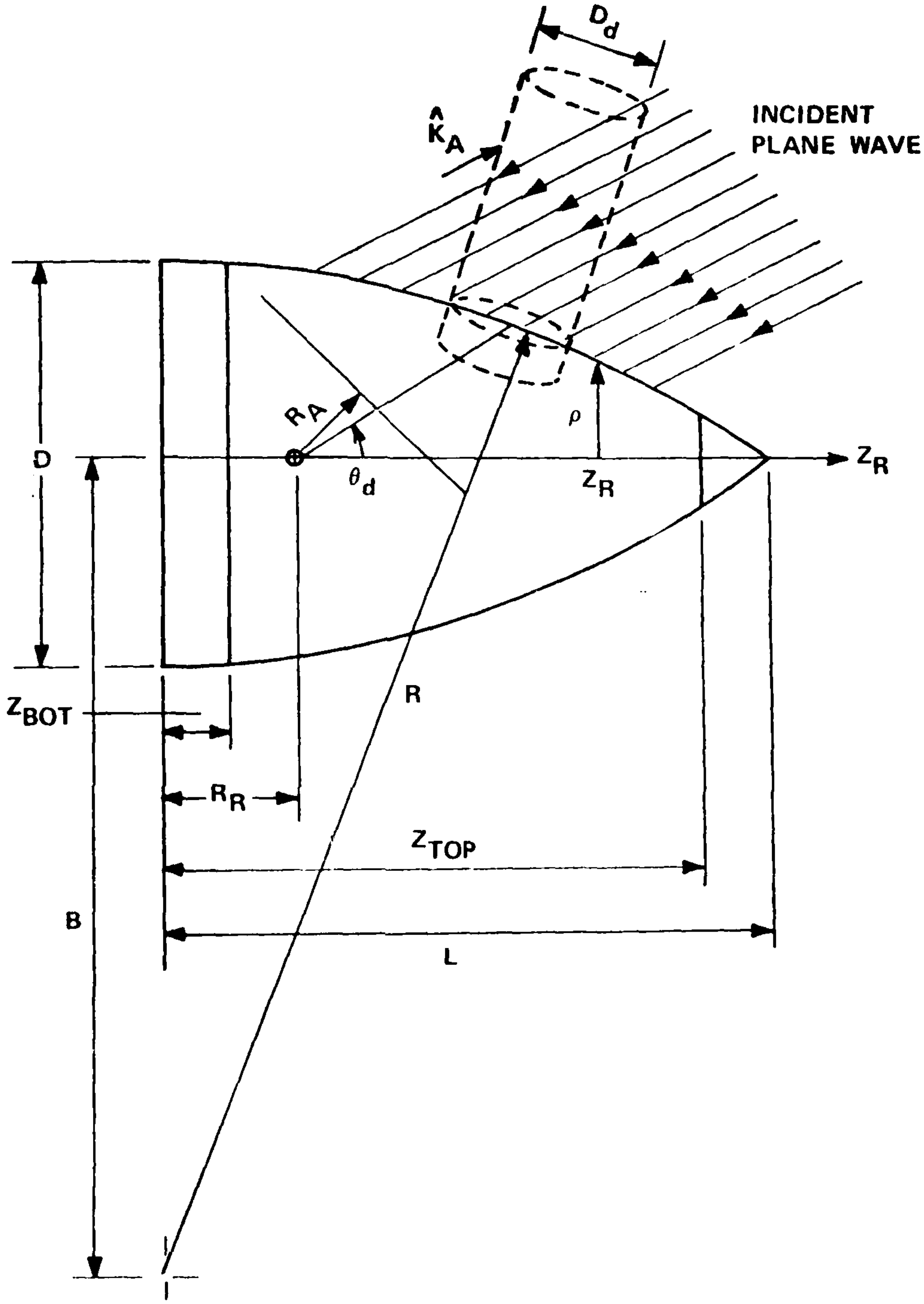

Figure 2-1. Tangent Ogive Radome Geometry.

and the dimension B is given by

$$B = R_{OS} - D_{OS}/2 \tag{6}$$

The placements of a bulkhead (bottom disk) and metal tip (top disk) can be specified by Z_{BOT} and Z_{TOP}, respectively. The thickness, dielectric constant, and loss tangent of the wall may also be specified for up to N=5 layers. The radome is assumed to be a body of revolution with uniform wall dimensions independent of location. The dashed cylindrical shape of a diameter D_d in Figure 2-1 was used earlier to simulate a laser-induced defect and is not pertinent here.

The subroutine which generates the antenna aperture fields represents two types of antennas: circular aperture with uniform illumination and any one of four polarizations (vertical, horizontal, RHC, LHC); flat plate antenna with tapered illumination and vertical polarization. For either antenna, the fields are computed for one of three selected channels: sum, azimuth difference, elevation difference. Inputs include the number of samples N_X, N_Y and the aperture diameter D_{AP}/λ in wavelengths.

The antenna/radome orientation is specified according to the parameters defined in Figure 2-2. The angle ϕ_P selects the plane of scan of the radome tip with respect to the antenna coordinate system: $\phi_P = 0°$ selects the azimuth plane; $\phi_P = 90°$ selects the elevation plane. The angle ϕ_L scans the tip in the selected plane.

The program computes boresight errors in the azimuth and elevation planes of the antenna. The radome orientation is specified by ϕ_P and ϕ_L. The first target return (plane wave) is made to arrive from the direction

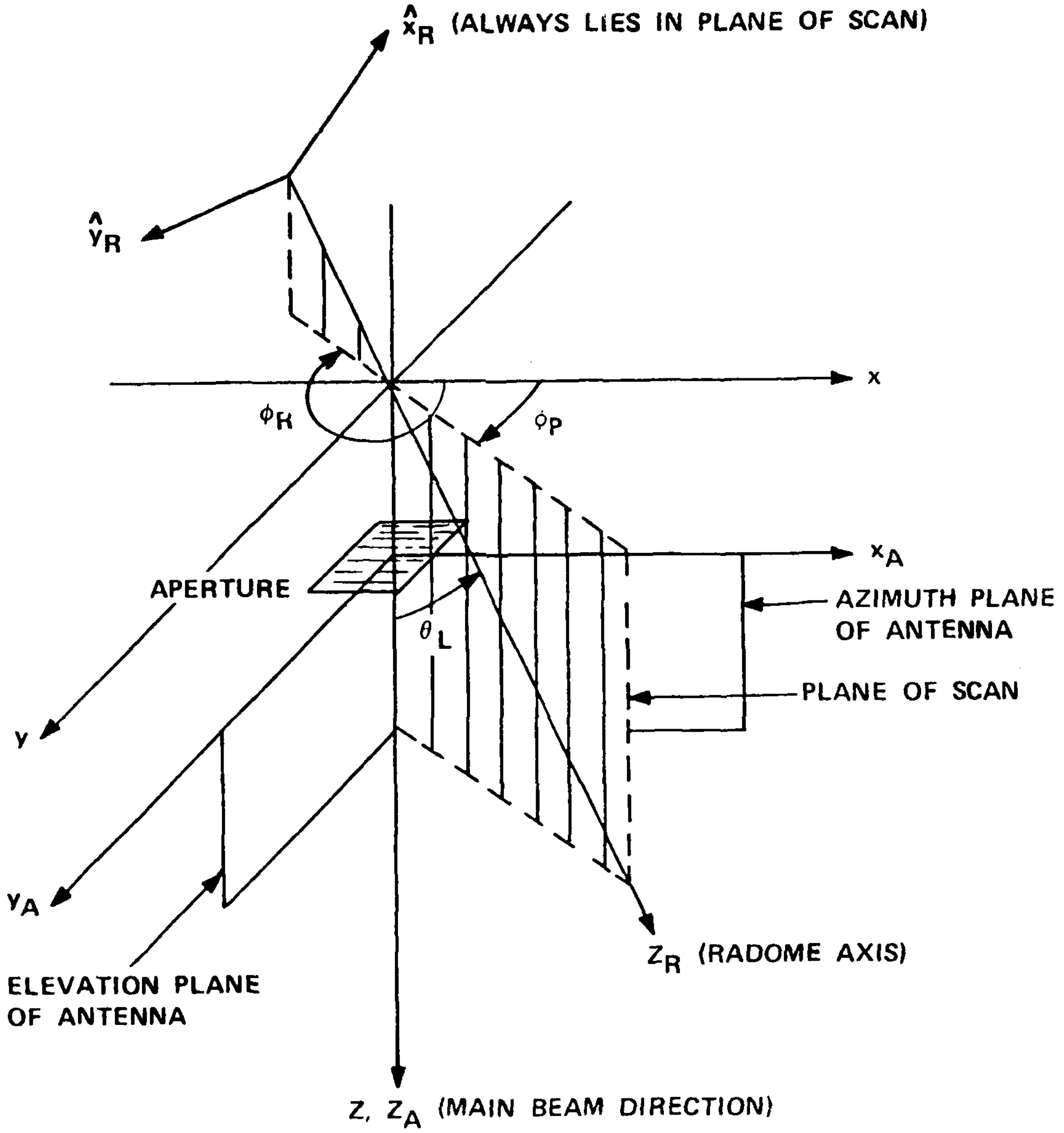

Figure 2-2. Coordinate Systems Used in Radome Analysis.

$$\hat{k}_1 = \hat{x}_A \sin\theta_{os} + \hat{y}_A \sin\theta_{os} + \hat{z}_A\sqrt{1 - 2\sin^2\theta_{os}} \tag{7}$$

where θ_{os} is the initial specified offset angle; e.g., 2°. The voltage received by each channel is computed and stored. The second return is made to arrive from

$$\hat{k}_2 = \hat{x}_A(-\sin\theta_{os}) + \hat{y}_A(-\sin\theta_{os}) + \hat{z}_A\sqrt{1 - 2\sin^2\theta_{os}} \tag{8}$$

and the voltages are again computed. The data from these two points are used to construct a linear tracking model in the two planes, and a direction of arrival $\hat{k}$ is predicted which will yield null indications in both planes. The process is repeated until a desired error tolerance is satisfied or a maximum number of iterations is exceeded. Upon completion, the output $\hat{k}$ indicates the direction from which the plane arrives which yields an electrical boresight indication. If α and β represent the boresight error angles in the azimuth and elevation planes, respectively, then they are related to the direction $\hat{k} = \hat{x}_A k_x + \hat{y}_A k_y + \hat{z}_A k_z$ by

$$\sin\alpha = \frac{k_x}{\sqrt{1 - k_y^2}} \tag{9}$$

$$\sin\beta = \frac{k_y}{\sqrt{1 - k_x^2}} \tag{10}$$

where

$$k_z = \sqrt{1 - k_x^2 - k_y^2} \quad . \tag{11}$$

Options are also provided whereby principal plane patterns as shown in Figure 2-3 and additional outputs around boresight can be computed and printed. These options are useful when preparing software for a new type of antenna and to ensure correct operation whenever curious results are obtained.

2-2. Usage:

Statement		Line No.
DATA APIN/0./		47
DATA ZBOTIN/0.00/		49
DATA RADIUS/1E0/		52
DATA THETAA, PHIA, AGAM3A/0.0,90.0,0.0/		53
DATA NX, NY, NXE, NYE, NXY/16,16,256,1,512/		56
DATA NREC, NS, MX, MY/32,16,16,1/		57
READ (5,6) TITLE		62
READ (5,*) GRAF3D, GRAFSA, GRAFTR, GRAFRV, SUPPRS, IPENCD		65
READ (5,*) NFINE, NPHI, NTHE, DIAOS, RA, RR, ZTOPIN, FREQ, OSANG		67
READ (5,*) LMAX, DMRAD, IOPT, RAPMAX, VAIRM, IPOL, ICASE, N, IPWR		76
READ (5,*) DIN(I), ER(I), TD(I)	(I=1,N)	108
READ (5,*) FINR(I)	(I=1,NFINE)	117
READ (5,*) PHI(I)	(I=1, NPHI)	120
READ (5,*) THETA(I)	(I=1,NTHE)	122

2-3. Arguments

a. Inputs. Units of arguments on input are distances in inches, angles in degrees, and frequency in gigahertz, unless otherwise noted. Units of arguments passed to subroutines are centimeters, radians, and

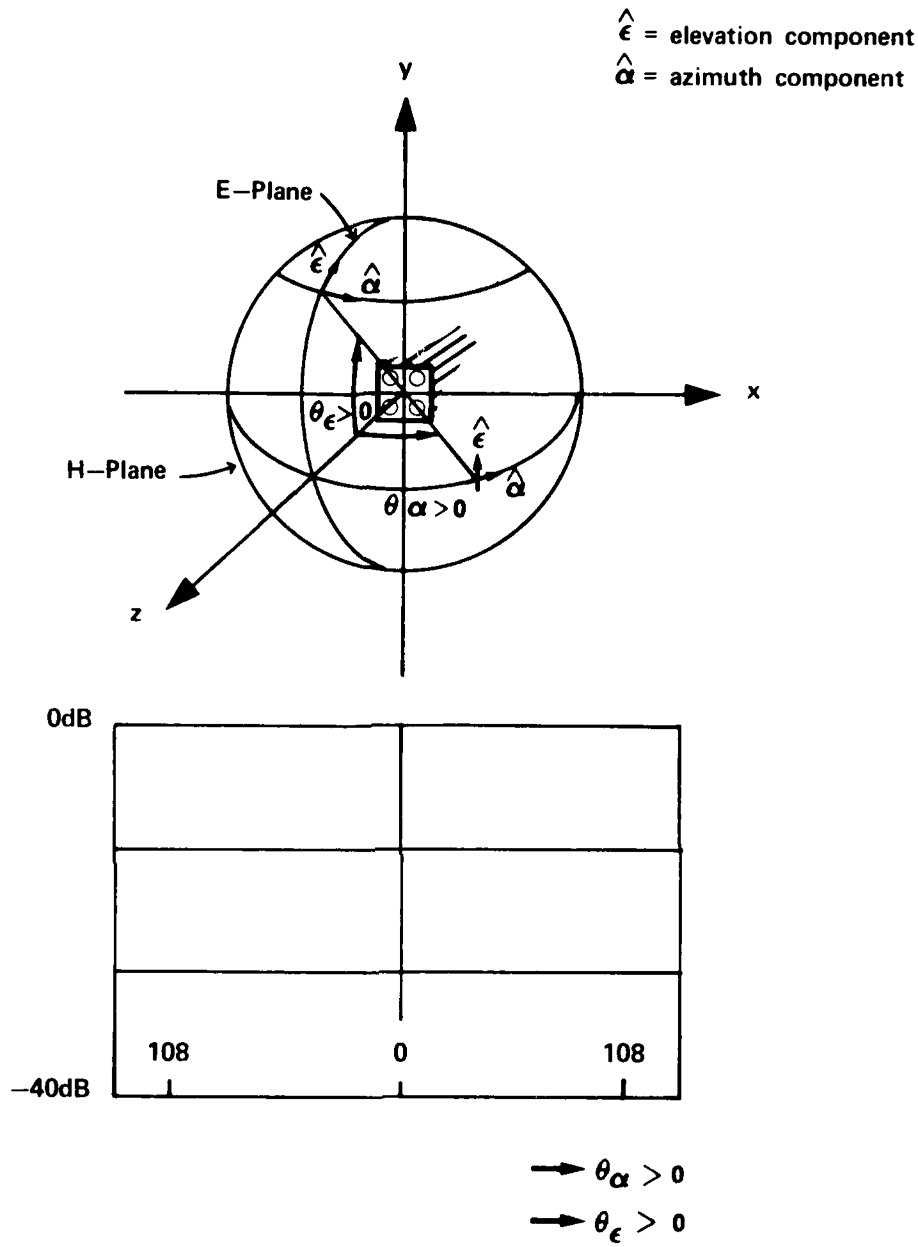

Figure 2.3 Coordinate System for Far Field Patterns

gigahertz. An asterisk is used to denote those DATA arguments that do not normally need to be changed by the user.

APIN* - Height of a cylindrical base section of the tangent ogive radome. It is no longer included in the ray tracing algorithms and should not be changed from its zero value.

ZBOTIN - Distance from base of tangent ogive radome to missile bulkhead (Figure 2-1).

RADIUS* - The radius R used in the far field factor e^{-jkR}/R by Subroutine FAR. Do not change.

THETAA* - Angle θ_a between z-axis and the position vector $\underline{r}_a$ to the antenna origin. This angle was used in earlier work to locate the antenna origin in the reference system using spherical coordinates (r_a, θ_a, ϕ_a). Do not change. See Chapter 7.

PHIA* - Angle ϕ_a between the projection of z_A axis onto the xy-plane and the x-axis. Do not change.

AGAM3A* - Angle between z_A-axis and z-axis in Figure 2-2. Do not change.

NX,NY - Integer powers of two equal to the number of sample points in the antenna aperture; e.g., 16, 32, 64, etc. Changing NX and NY necessitates compatible changes in Lines 16-18.

NXE,NYE - Integer powers of two which specify the expanded number of sample points desired when computing the transmitting patterns of the antenna by inverse Fourier transforming the aperture fields.

Subroutine JOYFFT provides this capability of increased resolution in one or both dimensions. Changes in NXE, NYE necessitate compatible changes in Lines 16, 20, 22, and 23. Note that NXE*NYE≤NX*NY and either NXE≤NX or NYE≤NY.

NXY - Integer power of two used by Subroutine JOYFFT for dimension of complex working array XYFFT. Note that MX*NX≤NXY and MY*NY≤NXY. See below for MX and MY.

NREC - Integer power of two equal to the number of points at which to compute the receiving pattern in either principal plane. The received voltage is computed at points θ_i equally spaced in sinθ, where θ is the angle measured from the z_A-axis as indicated in Figure 2-3, where sin θ_i = - KMAX + (I-1)*2*KMAX/NREC, and where KMAX = sin θ_{max} < 1.0.

NS - Not used. It was originally used by Subroutine RECBS. Do not remove.

MX,MY - Integer powers of two equal to the magnification factors desired in the k_x and k_y (E-plane) directions, respectively, of the transmitting antenna patterns. Note that the restrictions MX*NY≤NXY and MY*NY≤NXY must be observed. The data cited above indicated increased resolutions in the NX direction of MX=16 and no magnification (MY=1) in the NY direction. Consequently, note that NXE=MX*NX=256.

TITLE - A Hollerith string of up to 72 characters which describes briefly the analysis being done. A format of 18A4 is specified and should work for machines with word length greater than or equal to 32 bits. The dimension of TITLE (Line 31) should be at least 18.

GRAF3D - A logical variable used to control the plotting of the incident fields on the antenna aperture. This feature has been removed from the program, and GRAF3D should always be FALSE.

GRAFSA - A logical variable which (if TRUE) controls the plotting of the transmitting power patterns of the antenna as follows: E-plane sum, E-plane difference equation (Δ_{EL}), H-plane sum, and H-plane difference azimuth(Δ_{AZ}). The radome is absent.

GRAFTR - A logical variable which controls the plotting of the amplitude and phase of the antenna aperture fields in the following order:

$E_{X\Sigma}$, $E_{Y\Sigma}$, $E_{X\Delta EL}$, $E_{Y\Delta EL}$, $E_{X\Delta AZ}$, $E_{Y\Delta AZ}$.

GRAFRV - A logical variable which controls the plotting of the receiving patterns of the antenna with radome in the same order as specified under GRAFSA above.

SUPPRS - A logical variable which controls the printing of numerous results as illustrated in the test data in Section 2-6 below. When TRUE, the printing of these numerous results are suppressed. This feature

is convenient to aid in debugging new portions of software prior to making production runs.

IPENCD - An integer variable which selects pen and paper for the Calcomp. This variable may be system dependent. For the Cyber 70, IPENCD=00 yields ballpoint pen and 11" wide plain paper; IPENCD=40 yields a heavier ink pen and the same paper.

NFINE - Integer variable equal to the number of fineness ratios to be considered for the tangent ogive radome; e.g., NFINE=1.

NPHI - Integer variable equal to the number of scan planes; e.g., NPHI=2.

NTHE - Integer variable equal to the number of angles in each scan plane at which to compute boresight errors, etc. Note: The program is set up to iterate on fineness ratio, scan plane, and scan angle as outer loop, middle loop, and inner loop, respectively. Therefore, for each of NFINE fineness ratios, the analysis will be done for NTHE scan angles in NPHI different scan planes.

DIAB - Real variable equal to the outside base diameter (in.) of the radome. See Figure 2-1.

RA - Real variable equal to the distance (in.) from the gimbal point to the antenna aperture.

RR - Real variable equal to the distance (in.) from the gimbal point to the base of the radome.

ZTOPIN - Real variable equal to the distance (in.) from the base of the radome to the face of a metal tip on the radome.

FREQ - Real variable equal to the frequency of operation in gigahertz.

OSANG - Real variable equal to the offset angle in degrees at which the first target return is to arrive on the antenna; e.g., OSANG=3.0.

LMAX - Integer variable equal to the maximum number of iterations allowed by Subroutine RECBS in computing boresight error; e.g., LMAX=5.

DMRAD - Real variable equal to the tolerance in milliradians allowed on computing boresight error; e.g., DMRAD=0.1.

IOPT - Integer variable which selects the polarization of the incident plane wave as follows:

1. Linear, elevation component
2. Linear, azimuth component
3. Right hand circular
4. Left hand circular

RAPMAX - Real variable equal to the maximum radius (in.) of the antenna aperture. See Figure 3-1.

VAIRM - Real variable equal to the maximum amplitude of sum channel received voltage without radome. Any real value can be entered for this variable since a subsequent program modification (Lines 326-328) causes VAIRM to be computed automatically.

IPOL - Integer variable which selects the polarization of the antenna when ICASE=1 according to the

same code as used above for IOPT.

ICASE - Integer variable which selects one of two types of antenna apertures for the analysis: ICASE=1 or 2 selects a circular aperture with uniform illumination; ICASE=3 selects a flat plate antenna with programmed illumination. See Subroutine HACNF in Chapter 3.

N - Integer variable equal to the number of layers (up to 5) in the radome wall. For cases where more than 5 layers are required, the dimensional arrays on Line 37 must be changed to NN=N+1.

IPWR - Integer variable which selects the component for which to compute the transmitting power patterns as follows:

1. Elevation Component
2. Azimuth Component
3. Total power

DIN,ER,TD - Subscripted real variables equal to the thickness (in.), dielectric constant (ϵ_r), and loss tangent (tan δ) of each layer of the radome wall. I=1 corresponds to the first layer and is the layer on the exit side of the wall. Layer N is the first layer encountered by the incident plane wave. See Subroutine WALL.

FINR - Subscripted real variable equal to NFINE fineness ratios.

PHI - Subscripted real variable equal to NPHI angles (degrees) which specify the scan planes.

THETA - Subscripted real variable equal to NTHE angles (degrees) which specify the scan angles in the scan plane.

b. Outputs. The parameters of analysis which are computed and and outputted by the program depend on whether SUPPRS is true. In what follows, it is assumed that SUPPRS=FALSE so that all possible outputs are obtained. Since many of the original input parameters are printed directly, only those parameters not already explained above will be included below. Additional clarification may be found in Section 2-6.

TABLE - Logical variable which, if TRUE, causes a look-up table to be used in computing transmission coefficients. When SUPPRS=FALSE, an abbreviated table of transmission coefficients of the radome wall is printed by Subroutine WALL with variables as explained immediately below.

ANGLE - Real variable equal to the angle of incidence (degrees) of the plane wave on a plane sheet of infinite extent having the layered configuration specified for the radome wall. The entries in the table are computed at 250 equal increments in $\sin\theta_i$, but only every fifth result is printed.

TPERI,TPARI - Complex variables equal to the voltage insertion transmission coefficients of the sheet for the two cases of $\underline{E}_i$ perpendicular to the plane of incidence ($T_\perp$) and $\underline{E}_i$ parallel to the plane of incidence ($T_{||}$). In the printed table, the power transmission coefficients $|T_\perp|^2$ are

$|T_{||}|^2$ are printed; adjacent to each, the phases of $T_{\perp}$ and $T_{||}$ are also printed.

RPERP,RPARL- Complex variables equal to the reflection coefficients $R_{\perp}$, $R_{||}$ of the plane dielectric sheet. Actually, $|R_{\perp}|^2$ and $|R_{||}|^2$ are printed, accompanied by the phases $R_{\perp}$ and $R_{||}$.

KXMAX - Real variable equal to the folding wavenumber associated with sampling the aperture fields according to KXMAX = 1./2($\Delta x/\lambda$), where Δx is the distance between samples. See Subroutines HACNF and FFTA.

DXWL - Real variable equal to $\Delta x/\lambda$.

KXM,KYM - Real variables equal to the folding wavenumbers of the principal plane patterns after magnification for increased resolution. KXM=KYMAX*NXE/(MX*NX) and applies to the H-plane. KYM=KYMAX*NYE/(MY*NY) and applies to the E-plane. Usually, the expanded dimension NXE and magnification factor MX are selected so that KXM=KXMAX. Also, NYE and MY are usually selected so that KYM<<KYMAX.

MIN,MAX Real variables equal to the minimum and maximum values of the amplitude of the complex arrays containing the aperture fields as processed by Subroutine NORMH in preparation for 3D plotting by Subroutine PLT3DH.

ROS - Real variable equal to the radius of curvature of the outside shape of the tangent ogive radome.

BOS - Real variable equal to the distance B in inches defined in Figure 2-1.

FINOS - Real variable equal to the fineness ratio of the radome as based on the outside dimensions.

FINIS - Real variable equal to the fineness ratio of the radome as based on the inside dimensions.

The following variables are printed when the receiving patterns are computed and printed:

ICUT - Integer variable which defines the E-plane (ICUT=1) or H-plane (ICUT=2) pattern. See Figure 2-3.

ICOMP - Integer variable which defines the field component of the plane wave incident on the receiving antenna: ICOMP=1 for elevation component; ICOMP=2 for azimuth component.

KMAX - Real variable equal to the sine of the maximum angle off broadside for which the received voltage is computed.

NREC - Integer variable (power of 2) equal to the number of points at which the receiving pattern is computed. The pattern is computed at NREC points spaced equally in $k_{xy}=\sin\theta$ according to $\Delta k_{xy} = 2\ KMAX/NREC$.

DK - Real variable equal to 2*KMAX/NREC.

ANGMAX - Real variable equal to $\sin^{-1}$(KMAX).

The receiving pattern is computed at NRB points and magnified using Subroutine MAGFFT to 256 points equally spaced in $\sin^{-1}$ over the range (-KMAX, KMAX-DK). Three parameters are printed: angle in degrees, amplitude in decibels, and phase in degrees. Only every fourth point in the 256 points is printed. The receiving patterns are printed in the following order:

E-Plane : [illegible]$_{EL}$, [illegible]$_{EL}$

H-Plane : [illegible]$_{AZ}$, [illegible]$_{AZ}$

Subroutine RECM maintains a count NRAY of the number of rays actually traced from points in the aperture to the radome wall. When SUPPRS FALSE, this number will be printed.

Subroutine RECBS computes the boresight error of the antenna as produced by the radome. When SUPPRS FALSE, the following parameters are printed:

K1, K2 - Real subscripted variables containing the direction cosines (k_{xi}, k_{yi}, k_{zi}) of the last and next to last true directions to the target. One of these variables is equal to K, the subscripted variable containing the direction cosines of the last target return.

AZTM,ELTM - Real variables equal to the boresight error in the H-plane and E-plane associated with the last target return (k_x, k_y, k_z). Expressed in milliradians, these errors are computed according to

$$AZTM = \sin^{-1}(k_x/\sqrt{1-k_y^2})*1000.$$

$$ELTM = \sin^{-1}(k_y/\sqrt{1-k_x^2})*1000.$$

Let $\hat{k} = \hat{x}_A k_x + \hat{y}_A k_y + \hat{z}_A k_z$. Then AZTM is the angle between the z_A-axis and the projection of $\hat{k}$ onto the $x_A z_A$ (azimuth) plane. ELTM is the angle between the z_A-axis and the projection of $\hat{k}$ onto the $y_A z_A$ (elevation) plane.

MESAZ,MESEL - Real variables equal to the monopulse error slopes in the azimuth and elevation channels expressed in units of volts per degree, where the maximum signal received by the sum channel is considered to be one volt.

VAZ,VEL - Real subscripted variables equal to the received tracking functions I_{max} [illegible] corresponding to the target returns K1 and K2 above; e.g., VAZ(1) = I_{max} [illegible] for K1.

VMAX - Real variable equal to the maximum amplitude of the received sum channel voltage.

ICTR - Integer variable equal to the number of iterations (target returns) used by subroutine RFCBS to compute boresight error.

Subroutine RFCBS also computes and prints six additional monopulse outputs around the apparent boresight direction $\hat{k}_0$. The directions $\hat{k}$ chosen lie in the plane $k_x k_y$ and are spaced one milliradian apart over the range ±3 mrad and centered on the direction $\hat{k}_0$. The variables printed are as follows:

ANG - Real variable equal to the angle in milliradians between $\hat{k}_0$ and $\hat{k}$.

VRAZ,VREL - Real variables equal to I_{max} [illegible] for the target return from direction $\hat{k}$ for the azimuth and elevation channels, respectively.

DAZ,DEL - Amplitude and phase (degrees) of the complex voltages received on the Δ_{AZ} and Δ_{EL} channels, respectively, for target return k.

SLPAZ,SLPEL- Average values of the monopulse error slopes (volts/degree) in the azimuth and elevation channels, respectively, obtained by a linear approximation of the tracking functions based on their values at ANG = ±3 mrad. For example,

$$SLPAZ = [VRAZ(3\ mrad) - VRAZ(-3\ mrad)]/(.006*57.3)$$

The main program always prints the boresight error in azimuth (BSEAZ) and elevation (BSEEL), and the values printed are identical to AZTM and ELTM [illegible]. Main [illegible] prints the gain of the antenna in decibels with [illegible]

GAIN [illegible] * ALOG1[illegible]MAX VALUE

[illegible]", GAIN is negative and [illegible] radome reflections and [illegible] voltage, VAINA, is always printed [illegible] of the program.

[illegible]

[illegible] are presented in [illegible]

[illegible]

b. Supporting Subroutines. Thirty four supporting subroutines are required by RTFRACP. The purpose of each one is briefly described below.

(1) HACNF--Computes complex vector aperture electric fields of antenna for all three monopulse channels at NX x NY sample points.

(2) ORIENT--Computes matrices ROTATE and TRANSLate used for coordinate transformations by Subroutines POINT and VECTOR.

(3) POINT--Transforms a point $P(x_A, y_A, z_A)$ in antenna system to the same point $P(x_R, y_R, z_R)$ in radome coordinate system, and vice versa.

(4) VECTOR--Transforms a vector from radome to antenna coordinate system, and vice versa.

(5) INCPW--Computes the rectangular electric field components of a plane wave incident from the direction $\hat{k}_A$ in antenna coordinates. The power density of the plane wave is unity.

(6) RECM--Computes the voltage received by each channel of the antenna for a plane wave $PWI(E_x, E_y, E_z)$ incident on the radome from the direction $KA(k_x, k_y, k_z)$. Subroutine RECM calls the following subroutines:

VECTOR, POINT, TRACE, EXMIT, GANB.

(7) TRACE [illegible] and calls subroutines [illegible], [illegible], [illegible], [illegible], [illegible], [illegible], [illegible], [illegible], and [illegible].

(8) EXMIT [illegible]

[illegible]

dielectric wall with unit inner normal $\hat{n}$. The unit vectors $\hat{k}$, $\hat{n}$ are used to resolve the incident plane wave into vector components perpendicular and parallel to the plane of incidence, and to determine the angle of incidence. RXMIT calls Subroutines WALL and AMPHS.

(9) WALL--Computes the voltage insertion transmission coefficients of flat panel model of the radome wall as function of the sine of the incidence angle.

(10) AXB--Computes real vector cross product $\underline{C} = \underline{A} \times \underline{B}$.

(11) CAXB--Computes the complex vector cross product $\underline{C} = \underline{A} \times \underline{B}$.

(12) RECBS--Computes boresight errors of antenna enclosed by the radome for the specified orientation, fineness ratio, etc. RECBS calls Subroutines INCPW, RECM, and AMPHS.

(13) RECPTN--Computes receiving patterns of all three channels. RECPTN calls Subroutines INCPW and RECM.

(14) OGIVE--Computes point of intersection of ray and ogive by solving a quartic equation. OGIVE calls Subroutines CBRT, SQR, and XY.

(15) CBRT--Computes cube root.

(16) SQR--Computes square root with test for negative argument.

(17) OGIVEN--Computes the unit inward normal vector to the ogive surface at the point $P(x_R, y_R, z_R)$.

(18) XY--Used by Subroutine OGIVE to compute the x_R and y_R components of the point of intersection of a ray on the [illegible] surface.

(19) [illegible]--Computes the point of intersection of a ray and [illegible] representing the bulkhead inside the radome.

(20) BDISKN--Computes unit normal vector to bulkhead $(\hat{n} = +\hat{z}_K)$.

(21) TDISK--Computes the point of intersection of a ray and the base of the metal tip on the radome.

(22) TDISKN--Computes unit normal vector to metal tip $(\hat{n} = -\hat{z}_K)$

(23) FAR--Computes the amplitude of the power pattern from the complex plane wave spectra $A_x(k_x,k_y)$, $A_y(k_x,k_y)$ of an antenna.

(24) AMPHS--Converts a complex number from rectangular to polar form. This subroutine utilizes the intrinsic function ATAN2. The amplitude produced is linear (not decibels), and the phase is in degrees on the range (-180, 180).

(25) DBPV--Converts a real, two-dimensional array from linear to logarithmic values in decibels on the range 0 to -40 dB.

(26) NORMH--Normalizes a two-dimensional real array to values between 0 and 1.

(27) CNPLTH--Plots single dimensional far field patterns on axes patterned after standard pattern recorder paper. CNPLTH calls Subroutine PSI in addition to the usual Calcomp subroutines.

(28) PSI--Used by Subroutine CNPLTH to compute the azimuthal angle ψ.

(29) PLT3DH--Yields three-dimensional plots of the data in the two-dimensional real array FIELD. PLT3DH calls Subroutines PLTT, NORMH as well as the usual Calcomp subroutines.

(30) PLTT--Used by Subroutine PLT3DH to eliminate moving the pen for hidden lines.

(31) FFTA--Computes the Fast Fourier Transform of a one-dimensional complex array having 2**IEXP elements. Proper operation is machine dependent.

([illegible]) MAGFFT--Provides increased resolution of a sampled function using FFT and Discrete Fourier Transform techniques.

([illegible]) JOYFFT--Provides increased resolution of selected portions of two-dimensional Fourier transform. JOYFFT calls subroutines FFTA and PWRTWO.

([illegible]) PWRTWO--Used by subroutine JOYFFT to ensure that a given [illegible] power of [illegible].

[illegible] Program Flow

For the following, refer to the program listing in Section [illegible] and to line numbers shown on the right-hand margin of that listing.

Line No.	Explanation
Line [illegible]:	All variables beginning with the letter R in the main program are real.
Lines [illegible]:	[illegible] variables and array dimensions. Note equivalence statements in lines [illegible]. The dimension [illegible] in line [illegible] may be computer system dependent. Note in line [illegible] that only twenty [illegible] ratios, [illegible] planes, and mean angles can be accommodated.
Lines [illegible]:	Label common is used as a convenient means to transmit variables to subroutines not directly called by MAIN. The labels are generated from the names of the subroutines which receive the variables, and each label is terminated with the letter C to denote common; e.g., TDISKC denotes variables common to MAIN and subroutine TDISK.

Lines 40-42: Declare namelists for printing data. These namelists are no longer used except for occasional debugging purposes.

Lines 43-57: Set data in DATA statements as described above in Section 2-3.

Lines 61-62: Set SMAX and VMAX to unity to prevent division by zero.

Lines 63-64: Read and write TITLE according to 18A4 format.

Lines 65-67: Read input data using free-field format.

Line 68: Compute sine of the offset angle θ_{OS}.

Line 69: Set TABLE=FALSE so that normalizing factor VAIRM can be computed (Lines 319-329) via a call to Subroutines RECM and RXMIT. In the latter, TABLE=FALSE causes $T_{\perp}$, $T_{||}$ to be set to unity as in the case of no radome.

Lines 71-75: Write input data.

Lines 76-77: Read input data and set VAIRM needlessly.

Lines 78-104: Comments explaining input variables.

Line 105: Set NN=N+1= Number of wall layers plus one.

Line 106: Initialize DINCH= total thickness of radome wall in inches.

Lines 107-109: Read wall data and compute total thickness.

Line 110: Compute DIAIN= inside base diameter of the radome in inches.

Lines 111-112: Compute indices of the center element of near-field arrays corresponding to $x_A=y_A=0$.

Lines 113-114: Write array dimensional data.

Lines 115-122: Read fineness ratios, scan planes, and scan angles.

Lines 123-126: Compute wavelength in inches and centimeters. Compute $\beta=2\pi/\lambda_{cm}$.

Lines 127-128: Call RXMIT and compute table of transmission coefficients versus sine of incidence angle. The first call to RXMIT builds the table. Subsequent calls use the table if TABLE=TRUE.

Line 129: Compute DAPWL= diameter of antenna aperture in wavelengths.

Lines 130-139: Convert variables in inches to centimeters for input to subroutines. Some variables are multiply defined to avoid conflicts in labeled common; e.g., ZBOT and Z1. Note that DIACM is the inside diameter of the radome in centimeters.

Lines 140-144: Convert angles from degrees to radians using RAD=π/180.

Lines 145-151: Compute near fields of three channel monopulse antenna using Subroutine HACNF.

Lines 152-158: Set KYMAX=KXMAX, compute magnified folding wavenumbers KXM, KYM, and print results.

Lines 159-177: Initialize Calcomp plotter, if required. The commented initialization (Lines 164-174) applies to the IBM 3033 system at JHU/APL.

Note: Lines 178-258 are used to plot the near fields of the antenna and/or the transmitting principal plane power patterns.

Lines 178-179: Initialize the maximum values FMXEL, FMXDAZ of the E- and H-plane patterns so that when used initially as inputs to Subroutine FAR, the resulting pattern will be normalized with respect to its own maximum and FMXEL and FMXDAZ will be set equal to these respective maxima. On subsequent calls to FAR, the resulting patterns will be normalized with respect to FMXEL and FMXDAZ. Hence, the relative gain of the difference and sum patterns will be correctly displayed in the graphs.

Line 180: Iterate for each of three monopulse antenna channels.

Lines 181-190: Equate complex arrays EXT, EYT to the selected near field and compute the amplitude NF of EXT.

Line 191: Assume transmitting near fields are to be plotted (GRAFTR=T).

Line 193: Call Subroutine PLT3DH to plot the amplitude of EXT. The inputs XSIZE=6., YSIZE=2.5, HEIGHT=2.5 yield a 3D plot that will fit on a 8½" x 11" report page. The inputs NF, NX, NY specify the real array to be plotted and its dimensions. The input NMZ=.TRUE. directs the subroutine to normalize NF so that its values be between 0 and 1. The input LDB=.FALSE indicates that the array NF contains linear values rather than logarithmic values (decibels).

Lines 194-201: Compute and plot phase of EXT on a scale of -180 degrees to +180 degrees. Note that Line 199 ensures that the real array NF contains these phase values scaled to the required 0 to 1 range.

Lines 202-215: Repeat amplitude and phase 3D plots for EYT.

Line 216: Assume GFAFSA=T so that principal plane patterns are plotted.

Line 219: If IP=3, go to Line 243 and plot H-plane patterns; otherwise, plot E-plane patterns.

Line 222: Call Subroutine JOYFFT to calculate the inverse Fourier transform of the x_A-component of near field EXT to produce the plane wave spectrum XEEL from which the radiation field can be computed. In the process of computing the transform, provide increased resolution from NX x NY points to NYE x NXE points through the point (NXC,NYC) in the array EXT. In the k_x direction, the plane wave spectrum is magnified by MY; it is magnified by MX in the k_y direction. The array FFTXY is a working array.

Line 223: Repeat for EYT to produce the plane wave spectrum YEEL for the y_A-component of field.

Line 224: Call Subroutine FAR to calculate the E-plane elevation (IPWR=3) power pattern FFSEL of the near field at equal samples in $\sin\theta$ over the range (-KXM, KXM -ΔK). If FMXEL[illegible]0 (and it is for IP=1), normalize FFSEL with respect to its own maximum.

Line 226: Call Subroutine DBPV and convert the power pattern to decibels on a scale of 0 to -40 dB.

Lines 227-23[illegible]: Scale the values in FFSEL to the range of 0 to 1 for plotting.

Line 231: Call Subroutine CNPLTH and plot the power pattern. If KXM<1, the pattern is plotted over the angular range corresponding to $\sin^{-1}$(KXM); if KXM>1, the angular angle is (-90°, 90°). Subroutine CNPLTH actually plots conical cuts corresponding to k_x = constant or k_y = constant as specified by inputs KXC, KYC. In the call here, KXC=KYC=0 so that a principal pattern is produced.

Lines 232-236: Write a figure title for the plot and establish a new origin for the next plot.

Line 237: If IP=2, the E-plane patterns are finished.

Lines 238-242: Since JOYFFT changes the input arrays EXT,EYT, it is necessary to recompute them so that increased resolution can be obtained in the plane wave spectra in the H-plane.

Lines 243-258: Repeat computation and plotting for H-plane power patterns.

Line 260: Iterate the radome analysis for NFINE fineness ratios.

Line 261: Set FINE = outside fineness ratio.

Lines 262-266: Calculate and write R_{OS}, B, F_{OS}, F_{IS} as defined in Figure 2-1 for the radome geometry.

Line 267: Compute RDML = distance from the base of the radome to the theoretical tip on the inside of the radome.

Lines 268-272: If ZTOPIN<RDML, the radome has a metal tip, and a message is written to that effect.

Lines 273-283: Compute parameters needed by Subroutine OGIVE to describe the radome shape. R and B are in centimeters

and apply to the inside dimensions. AP, the height of the cylinder in centimeters, is not used. RTSQ= square of the radius of the top disk. RBSQ= square of the radius of the bottom disk (bulkhead). The other variables, BSQ, RINV, RSQ1, RP, and RP2, are precalculated here to speed later computations in OGIVE.

Line 285: Compute conversion factor DPMR for converting milliradians to degrees.

Lines 286-288: Initialize the "last" values of boresight error in azimuth (AZL) and elevation (ELL) and the "last" value THL of scan angle. These variables are used later to compute boresight error slope in degrees per degree from the present and last values of boresight error.

Lines 289-290: Write title for analysis results.

Lines 291-293: Write parameters of radome wall.

Lines 294-296: Write heading for table of boresight error and gain data.

Lines 297-301: Write this same data to logical unit 7 for subsequent storage as a disk file, if desired.

Line 309: Iterate the radome analysis for NPHI scan planes.

Lines 310-312: Compute ϕ_r in radians as required by Subroutine ORIENT.

Line 313: Iterate the analysis for NTHE scan angles in each scan plane.

Lines 314-316: Compute θ_r in radians as required by Subroutine ORIENT.

Line 317: Call Subroutine ORIENT and compute the rotation matrix ROTATE and translation matrix TRANSL required for coordinate transformations using Subroutines POINT and VECTOR.

Line 318: On the first iteration, TABLE is false so that the maximum amplitude of the received voltage on the sum channel is computed without the radome.

Line 319-322: Set the direction cosines of the incident plane wave so that it arrives from the $\hat{z}_A$ direction.

Line 323: Call Subroutine INCPW and compute the rectangular components PWI of the incident plane wave having polarization specified by IOPT.

Lines 324-325: Set TSUP=T and TABLE=F so that an air radome wall be used and so that printing by Subroutines RXMIT and RECM will be suppressed.

Lines 326-327: Call Subroutine RECM and compute the complex voltages VR received on the sum, difference elevation, and difference azimuth channels, respectively, corresponding to VR(I), I=1,3.

Line 328: Compute VAIRM=|VR(1)|.

Line 329: Set TABLE=T so that on subsequent iterations VAIRM will not be recomputed, and so that the table of transmission coefficients will be utilized when RXMIT is called.

Line 330: If SUPPRS=F, compute and print the E-plane and H-plane receiving power patterns of the antenna with the radome in place.

Lines 333-334: Iterate in J for E-plane (ICUT=1) and H-plane (ICUT=2) patterns.

Line 335: Set the desired far field component.

Lines 336-337: Set KMAX=$\sin^{-1}(\theta_{max})$=.996. If KXMAX, as computed by HACNF, is less than KMAX, then use the smaller as the maximum angle in the principal plane at which to compute the pattern.

Line 338: Set the temporary logical variable TSUP=T so that printing will be suppressed.

Lines 339-340: Call Subroutine RECPTN and compute the complex received voltages on each of three channels at NREC points over the range (-KMAX, KMAX - DK).

Lines 341-344: Increase the resolution and print results for all three channels. Do not print results that are known to be identically zero.

Lines 345-346: Transfer the received voltage into a one-dimensional array VREC.

Line 347: If NREC>NXE, there is no need to increase the resolution.

Line 348: Call Subroutine MAGFFT to increase the resolution of VREC from NREC points to NXE points. The result is contained in complex array XYFFT on output.

Lines 349-353: Compute linear power pattern.

Line 354: Select NXX=larger of NXE and NREC.

Lines 355-356: Write heading for printed results from Subroutine NORMH.

Line 357: Call Subroutine NORMH to normalize the NXX values in real array MVREC to be between zero and one. The input argument LDB=.FALSE. since the values are not in decibels.

Line 358: Call Subroutine DBPV to convert the power pattern in MVREC to decibels.

Lines 359-360: Write correct heading for E-plane or H-plane.

Line 361: Compute the increment in $\sin\theta$ at which the power pattern has been computed and resolved.

Lines 362-368: Scale the power pattern to have values between 0 and 1. If SUPPRS=F, compute the angle θ ANG and the phase of the pattern, and print the results for every fourth angle.

Line 372: If GRAFRV=T, plot the receiving power patterns.

Lines 373-378: Call Subroutine CNPLTH and plot the receiving patterns in turn. Write an appropriate figure title following each pattern plot. Re-origin the plotter pen for subsequent plots. The result of Lines 330-383 is four principal plane patterns: E-plane sum, E-plane Δ_{EL}, H plane sum, H-plane Δ_{AZ}.

Lines 384-386: Call Subroutine PECBS and compute the boresight errors AZT, ELT in the azimuth and elevation planes of the antenna as caused by the radome. On output, the real array KA contains the direction cosines of the last target return and, hence, gives the true direction to the target at the time that the tracking functions in the azimuth and elevation planes indicated the electrical boresight direction.

Line 387: If this is the first iteration in scan angle, do not attempt to compute boresight error slope.

Lines [illegible]: Compute boresight error slope (degrees/degree) in azimuth and elevation channels.

Lines [illegible]: Set the "last" values of boresight errors and scan angle to the current values in preparation for next iteration.

Line 393: Compute loss in maximum gain of the antenna sum channel due to the radome.

Lines 394-39[illegible]: Write results to logical units 6 and 7.

Lines 399-400: Write maximum amplitude of received sum voltage VAIRM without radome.

Line 401: Terminate plotting software.

STOP

END

2-6. Test Cases

Four test cases are presented in Appendices A, B, C, and D to demonstrate correct operation of the radome analysis computer program ETTRACP.

Appendices A and B present the test data and results for a circularly (RHC) polarized antenna and five-layer tangent ogive radome at a frequency of 11.80285GHz (λ=1.0 inch). The diameter of the aperture is 11.84λ. The outside diameter of the radome is 16.267 inches. The fineness ratio is 3.00. In Appendix A, the program is exercised without plotting, and printing is minimized. In Appendix B, all plotting and printing options are exercised.

Appendices C and D present the test data and results for a vertically polarized flat plate antenna of diameter 5.1992λ. All other parameters of the analysis are the same as in Appendices A and B. Appendix C contains the

[illegible] Appendix [illegible]

[illegible]

In Appendix [illegible]

[illegible] the order that [illegible]

[illegible] dimensional [illegible].

Reference to Appendix A, [illegible] seen that [illegible] both planes over the entire [illegible] measurement [illegible] [illegible] are produced only in the plane [illegible] consideration. Further [illegible] antenna shows that depolarization of the incident plane wave [illegible] produces additive errors, and the results shown are [illegible] expectations.

The transmitting and receiving patterns in Appendix B [illegible] are not in agreement contrary to expectations. The discrepancy is due to the fact that the receiving patterns have a (1 + [illegible]) variation characteristic of the geometrical optics approximation used for H_T. On the other hand, the transmitting patterns have a [illegible] variation as characteristic of an assumption of only magnetic current sources in the aperture. The disagreement is significant only for angles away from boresight.

[illegible] time required for each test case on the Cyber [illegible] was as follows:

[illegible]	Time [illegible]	No. of Scan Angle
[illegible]		[illegible]
[illegible]		1
[illegible]		[illegible]
[illegible]		

[illegible] plane wave [illegible]

[illegible]

[illegible] Radome [illegible] Agreement

[illegible] Newton, "Parametric [illegible] Symposium [illegible] Symposium [illegible]

[illegible] Performance [illegible] DAAG-01-[illegible]

```
C  THIS RAY TRACING FORMULATION RADOME ANALYSIS COMPUTER PROGRAM,
C  RTFRACP, USES GEOMETRICAL OPTICS AND LORENTZ RECIPROCITY
C  TO COMPUTE THE RESPONSES AND BORESIGHT ERRORS OF A MONOPULSE
C  ANTENNA INSIDE A TANGENT OGIVE RADOME TO AN INCIDENT PLANE
C  ELECTROMAGNETIC WAVE OF SPECIFIED POLARIZATION (TARGET RETURN).
C
C  RTFRACP WAS DEVELOPED AT GEORGIA INSTITUTE OF TECHNOLOGY, ATLANTA
C  GEORGIA, PRIMARILY UNDER GRANT AFOSR-77-3469 (PHYSICS DIRECTORATE)
C  AND DOCUMENTED UNDER JHU/APL CONTRACT NO. 601053 (ROBERT C.
C  HALLALIEU) FOR USE ON A SIGNIFICANT RADOME TECHNOLOGY PROGRAM
C  FOR THE DEPARTMENT OF THE NAVY.
C
C  THIS VERSION IS FOR EXECUTION ON CYBER 70/74.  WITH ONLY MINOR
C  SYNTAX CHANGES,  IT HAS BEEN IMPLEMENTED ON THE IBM 3033 AT JHU/APL.
C
      PROGRAM RTFRACP(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7)
      IMPLICIT REAL(K)
      REAL NF(16,16),NVREC(256),KA(3)
      COMPLEX SUMX(16,16),SUMY(16,16),DELX(16,16),DELY(16,16)
      COMPLEX DAZX(16,16),DAZY(16,16),EXT(16,16),EYT(16,16)
      COMPLEX VP(16),VREC3(32,3),VREC(32)
      REAL FFS(256,1),FFSEL(1,256),NORM(3),P1(3)
      COMPLEX PAT(3),PWI(3)
      COMPLEX XE(256,1),YE(256,1),XYFFT(512)
      COMPLEX XEEL(1,256),YEEL(1,256)
      EQUIVALENCE(XE(1,1),XEEL(1,1))
      EQUIVALENCE(YE(1,1),YEEL(1,1))
      EQUIVALENCE(FFS(1,1),MVREC(1),FFSEL(1,1))
      LOGICAL GRAF3D,GRAFSA,GRAFTP,GRAFRV,TABLE,SUPPRS,TSUP
      INTEGER IBUF(512)
      REAL ROTATE(3,3),TRANSL(3),TITLE(18)
      REAL FINP(20),PHI(20),THETA(20)
C
      COMMON/TDISKC/ZTOP,RTSQ
      COMMON/TRACC/Z2,Z1
      COMMON/RDISKC/ZROT,RRSQ
      COMMON/TRANSC/DIN(6),ER(6),TD(6),TZ,WALTOL,N,NN,D(6),ZB,TK
      COMMON/OGIVC/PP,RSQ,AP,PINV,B,RSQ1,RP2
```

```
C
      NAMELIST/GEOM/RR,RA,AFIN,ZBOTIN,NX,NY,NXE,NYE,NXY,MX,MY,NXC,NYC
      NAMELIST/KDATA/KXMAX,KYMAX,KXM,KYM
      NAMELIST/NEW/LMAX,DMRAD,IOPT,RAPMAX,VAIRM
C  BOUNDARY VALUES NEEDED BY SUBR TRACE (INCHES, CONVERT TO CM BELOW)
C Z1=ZR COORDINATE OF BOTTOM DISK
C Z2=ZR COORDINATE OF TOP DISK (Z1,Z2 IN CM)
C  APIN IS HEIGHT OF CYLINDER IN INCHES, CONVERT TO CM BELOW
      DATA APIN/0./
C  ZBOTIN IS ZR COORD OF BOTTOM DISK (BULKHEAD) IN RADOME COORD IN INCHE
      DATA ZBOTIN/0.00/
C  KXMAX,KYMAX ARE OUTPUTS OF NEAR FIELD SUBR
C  INITIALIZE CONSTANTS
      DATA RADIUS/150/
      DATA THETAA,PHIA,AGAM3A/0.0,90.0,0.0/
      DATA PI/3.14159265358 98/
C************
      DATA NX,NY,NXE,NYE,NXY/16,16,256,1,512/
      DATA NREC,NS,MX,MY/32,16,16,1/
C************
C
C  READ IN DESCRIPTION OF RADOME WALL
      SMAX=1.0
      VMAX=1.0
      READ(5,6)TITLE
      WRITE(6,6) TITLE
      READ(5,*) GRAF3D,GRAFSA,GRAFTR,GRAFRV,SUPPRS,IPENCO
  260 FORMAT(4L6)
      READ(5,*) NFINE,NPHI,NTHE,DIAUS,RA,RR,ZTOPIN,FREQ,OSANG
      SINCS=SIN(CSANG*PI/180.)
      TABLE=.FALSE.
C  TABLE IS SET FALSE SO THAT NORMALIZING FACTOR CAN BE COMPUTED.
      WRITE(6,265) GRAF3D,GRAFSA,GRAFTR,GRAFRV,TABLE
  265 FORMAT(" GRAF3D=",L2," GRAFSA=",L2," GRAFTR=",L2," GRAFRV=",L2,
     & "  TABLE=",L2)
      WRITE(6,270) NFINE,NPHI,NTHE,CSANG
  270 FORMAT(/" NFINE=",I5," NPHI=",I3,"  NTHETA=",I3,"  CSANG= ",F5.2/)
      READ(5,*) LMAX,DMRAD,IOPT,RAPMAX,VAIRM,IPOL,ICASE,N,IPWR
```

```
      IF (VAIRM.LE.0.) VAIRM=1.0
C DIAOS=OUTSIDE DIAMETER OF BASE OF TANGENT OGIVE RADOME
C VAIR=MAXIMUM REC"D VOLTAGE W/O RADOME AT KX=0.,KY=0.
C NFINE=NO. OF FINENESS RATIOS
C  NPHI=NUMBER OF SCAN PLANES
C  NTHE=NUMBER OF ANGLES IN EACH SCAN PLANE
C DIAIN=INSIDE BASE DIAMETER OF RADOME IN INCHES
C ZTOPIN=ZR COORD (IN) OF TOP DISK (METAL TIP)
C FREQ=FREQUENCY IN GHZ
C GRAF3D=.TRUE. GIVES 3D PLOTS OF INCIDENT FIELDS ON APERTURE (DELETED)
C GRAFRV=.TRUE. GIVES SA PLOTS OF RECEIVING PATTERNS (AZ & EL)
C GRAFSA=.TRUE. GIVES SA PLOTS OF TRANSMITTING PATTERN WITHOUT RADOME
C SUPPRS=.TRUE. SUPPRESSES THE PRINTING OF NUMEROUS RESULTS
C RAPMAX=MAX RADIUS OF ANTENNA APERTURE IN INCHES.
C  IOPT SELECTS POLARIZATION OF INCIDENT PLANE WAVE:
C       =1 ELEV (VERTICAL)
C       =2 AZIMUUTH (HORIZONTAL)
C       =3 RHC
C       =4 LHC
C  IPOL SELECTS POLARIZATION OF ANTENNA WHEN ICASE=1:
C       = SAME CODE AS FOR IOPT
C  ICASE=1 OR 2 FOR CIRC APERTURE, UNIFORM ILLUMINATION
C        =3 FOR FLAT PLATE WITH SPECIFIED ILLUM, VERT POL (CASE III)
C  N=NUMBER OF LAYERS IN RADOME WALL
C  OSANG=ANGLE IN DEG IN 45 PLANE OFF BORESIGHT OF FIRST TARGET RETURN
C        USED BY SUBR RECBS IN GETTING INITIAL DATA.
C  IPWR=1 FOR POWER IN ELEV COMP OF FAR FIELD PATTERN
C       =2 FOR AZIMUTH COMP,=3 FOR TOTAL POWER.
      NN=N+1
      DINCH=0.
      DO 5 I=1,N
      READ(5,*) DIN(I),ER(I),TD(I)
    5 DINCH=DIN(I)+DINCH
      DIAIN=DIAOS-DINCH*2.
      NXC=NX/2+1
      NYC=NY/2+1
      WRITE(6,4) NX,NY,NXE,NYE,NXY,MX,MY
    4 FORMAT(" NX,NY,NXE,NYE,NXY,MX,MY:",7I4)
```

```
C READ FINENESS    IOS FOR THIS RUN--BASED ON OUTSIDE DIMENSIONS
      DO 13 I=1,NFINE
   13 READ(5,*) FINE(I)
C READ ORIENTATIONS FOR THIS RUN (DEGREES)
      DO 14 I=1,NPHI
   14 READ(5,*) PHI(I)
      DO 15 I=1,NTHE
   15 READ(5,*) THETA(I)
C  COMPUTE WAVELENGTH:
      WLIN=29.97925/(FREQ*2.54)
      WLCM=WLIN*2.54
      BETA=2.*PI/WLCM
C  INITIALIZE TABLE OF XMN COEFFICIENTS:
      CALL RXMIT(FWI,FWT,KA,NORM,P1,TABLE,SUPPRS,BETA)
      DAPWL=2.*RAPMAX/WLIN
C  CONVERT TO CENTIMETER AND RADIANS
      ZBOT=ZBOTIN*2.54
      Z1=ZBOT
      RSQMAX=(2.54*RAPMAX)**2
      ZTOP=ZTOPIN*2.54
      ZB=ZTOP
      Z2=ZTOP
      RA=RA*2.54
      RR=RR*2.54
      DIACM=DIAIN*2.54
      RAD=PI/180.0
    6 FORMAT(18A4)
      THETAA=THETAA*RAD
      PHIA=PHIA*RAD
      AGAM3A=AGAM3A*RAD
C  COMPUTE FIELDS OF ANTENNA WHEN XMITTING:
      CALL HACNF(SUMX,NX,NY,1,IPOL,1,DAPWL,DXWL,KXMAX,ICASE)
      CALL HACNF(SUMY,NX,NY,1,IPOL,2,DAPWL,DXWL,KXMAX,ICASE)
      CALL HACNF(DELX,NX,NY,2,IPOL,1,DAPWL,DXWL,KXMAX,ICASE)
      CALL HACNF(DELY,NX,NY,2,IPOL,2,DAPWL,DXWL,KXMAX,ICASE)
      CALL HACNF(DAZX,NX,NY,3,IPOL,1,DAPWL,DXWL,KXMAX,ICASE)
      CALL HACNF(DAZY,NX,NY,3,IPOL,2,DAPWL,DXWL,KXMAX,ICASE)
      KYMAX=KXMAX
```

```
      KXM=KXMAX*NXE/MX/NX
      KYM=KYMAX*NYE/MY/NY
      WRITE(6,3) KXMAX,DXWL,KXM,KYM
    3 FORMAT(" KXMAX=KYMAX=",F8.5,"  XY SPACING=",
     $F8.5," WAVELENGTHS"/"  KXM=",F6.5,"  KYM=",F8.5)
C
C  INITIALIZE PLOTTER SOFTWARE
      IF (GRAF3D.OR.GRAFSA.OR.GRAFTR.OR.GRAFRV) GO TO 200
      GO TO 205
  200 CONTINUE
C
C------------------- CALCOMP INITIALIZATION --------------------------
C
C     CALL TITL36("RADOME ANALYSIS COMPUTER PROGRAM",
C    *            "               G.K. HUDDLESTON      ",
C    *            " GEORGIA INSTITUTE OF TECHNOLOGY"  )
C     CALL INIT36( MDAY )
CC
C     CALL PLOT(0.,-3.,-3)
      CALL PLOTS(IBUF,512,3,IPENCD)
C
C--------------------------------------------------------------------
C
      IF (GRAFTR.OR.GRAFSA) GO TO 201
      GO TO 205
  201 FMXEL=0.
      FMXDAZ=0.
      DO 30 IP=1,3
      DO 35 I=1,NX
      DO 35 J=1,NY
      IF (IP.EQ.1) EXT(I,J)=SUMX(I,J)
      IF (IP.EQ.1) EYT(I,J)=SUMY(I,J)
      IF (IP.EQ.2) EXT(I,J)=DELX(I,J)
      IF (IP.EQ.2)EYT(I,J)=DELY(I,J)
      IF (IP.EQ.3) EXT(I,J)=DAZX(I,J)
      IF (IP.EQ.3) EYT(I,J)=DAZY(I,J)
      NF(I,J)=CABS(EXT(I,J))
   35 CONTINUE
```

```
      IF (.NOT.GRAFTR) GO TO 215
C PLOT 3D NEAR FIELDS X-COMPONENTS
      CALL PLT3DH(6.,2.5,2.5,NF,NX,NY,.TRUE.,.FALSE.)
C  PLOT PHASE ALSO
      DO 40 I=1,NX
      DO 40 J=1,NY
      NF(I,J)=0.
      CALL AMPHS(EXT(I,J),RLF,AIF)
      NF(I,J)=(AIF+180.)/360.
   40 CONTINUE
      CALL PLT3DH(6.,2.5,2.5,NF,NX,NY,.FALSE.,.FALSE.)
C PLOT 3D NEAR FIELDS Y-COMPONENTS
      DO 45 I=1,NX
      DO 45 J=1,NY
      NF(I,J)=CABS(EYT(I,J))
   45 CONTINUE
      CALL PLT3DH(6.,2.5,2.5,NF,NX,NY,.TRUE.,.FALSE.)
C  PLOT PHASE ALSO
      DO 50 I=1,NX
      DO 50 J=1,NY
      NF(I,J)=0.
      CALL AMPHS(EYT(I,J),RLF,AIF)
      NF(I,J)=(AIF+180.)/360.
   50 CONTINUE
      CALL PLT3DH(6.,2.5,2.5,NF,NX,NY,.FALSE.,.FALSE.)
      IF (GRAFSA) GO TO 215
      GO TO 30
  215 CONTINUE
      IF (IP.EQ.3) GO TO 220
C CALC EL CUT OF SUM
C  NOTE THAT  JOYFFT CHANGES EXT,EYT.
      CALL JOYFFT(EXT,NX,NY,MY,MX,NXC,NYC,XEEL,NYE,NXE,XYFFT,NXY,3)
      CALL JOYFFT(EYT,NX,NY,MY,MX,NXC,NYC,YEEL,NYE,NXE,XYFFT,NXY,3)
      CALL FAR(FFSEL,XEEL,YEEL,NYE,NXE,FREQ,KYM,KXM,RADIUS,IPWR,FMXEL)
C SA PLOTS OF ELEVATION RESULTS
      CALL DRPV(FFSEL,NYE,NXE,1)
      DO 216 I=1,NYE
      DO 216 J=1,NXE
```

```
      FFSEL(I,J)=1.0+FFSEL(I,J)/40.
  216 CONTINUE
      CALL CNPLTH(FFSEL,NXE,KXM,0.,0.)
      CALL SYMBOL(.5,6.5,.14000,39HFIGURE     TRANSMITTING ELEVATION PO
     $WER,0.,39)
      RPWR=FLOAT(IPWR)
      CALL NUMBER(999.,999.,.14,RPWR,0.,0)
      CALL PLOT(8.5,0.,-3)
      IF (IP.EQ.2) GO TO 30
C  RECOMPUTE SUMX,SUMY FOR JOYFFT!
      CALL HACNF(EXT,NX,NY,1,IPOL,1,DAPWL,DXWL,KXMAX,ICASE)
      WRITE(6,219) IPWR
  219 FORMAT(" IPOWER OF PATTERN=",I2)
      CALL HACNF(EYT,NX,NY,1,IPOL,2,DAPWL,DXWL,KXMAX,ICASE)
  220 CALL JOYFFT(EXT,NX,NY,MX,MY,NXC,NYC,YE,NXE,NYE,XYFFT,NXY,3)
      CALL JOYFFT(EYT,NX,NY,MX,MY,NXC,NYC,YE,NXE,NYE,XYFFT,NXY,3)
      CALL FAR(FFS,XE,YE,NXE,NYE,FREQ,KXM,KYM,RADIUS,IPWR,FMXDAZ)
C SA PLOTS OF AZIMUTH RESULTS
      CALL DBPV(FFS,NXE,NYE,1)
      DO 10 I=1,NXE,1
      DO 10 J=1,NYE
      FFS(I,J)=1.0+FFS(I,J)/40.0
   10 CONTINUE
      CALL CNPLTH(FFS,NXE,KXM,0.,0.)
  226 CALL SYMBOL(.5,6.5,.14000,37HFIGURE     TRANSMITTING AZIMUTH POWER
     $,0.,37)
      CALL NUMBER(999.,999.,.14,RPWR,0.,0)
      CALL PLOT(8.5,0.,-3)
   30 CONTINUE
  205 CONTINUE
C
      DO 100 NG=1,NFINE
      FINE=FINR(NG)
C CALCULATE INSIDE FINENESS RATIO
      RIN=FINE*DIAOS/(SIN(PI-2.*ATAN(2.*FINE)))
      BIN=RIN-DIAOS/2.
      FINE=SQRT((RIN-DINCH)**2-BIN**2)/DIAIN
      WRITE(6,20) RIN,BIN,FINR(NG),FINE
```

```
      ROML=FINF*DIAIN+APIN
      IF (ZTOPIN.LT.ROML) WRITE(6,25) ZTOPIN
   20 FORMAT(" TANGENT OGIVE PARAMETERS: "," ROS(IN)="
     $ ,F9.5," BOS(IN)=",F9.5,/26X," FINOS=",F5.3,
     $ " FINIS=",F8.5)
   25 FORMAT(/" THIS RADOME HAS A TOP DISK AT ZTOPIN= ",E12.5/)
C  COMPUTE PARAMETERS NEEDED BY SUBR OGIVE
      R=FINF*DIACM/(SIN(PI-2.*ATAN(2.*FINE)))
      B=R-DIACM/2.
      AP=APIN*2.54
      RTSQ=(SQRT(R**2-(ZTOP-AP)**2)-B)**2
      RBSQ=(SQRT(R**2-(ZBOT-AP)**2)-B)**2
      BSQ=B**2
      RINV=1./R
      RSQ1=R**2
      RF=RSQ1-BSQ
      RP2=RSQ1+BSQ
C
      DPMR=180./(PI*1000.)
      AZL=0.
      ELL=0.
      THL=0.
      WRITE(6,2) TITLE,FINR(NG),DIACS,ZTOPIN,FREQ,RA,RR,DAPWL,IPOL,
     $ICASE,IOPT
      DO 8 I=1,N
    8 WRITE(6,7) I,CIN(I),ER(I),TD(I)
    7 FORMAT(2X,I3,F13.5,F10.3,F9.4)
      WRITE(6,9)
    9 FORMAT(//"  PHI   THETA   BSEEL    BSEAZ    SLPEL      SLPAZ      GAIN"/
     $      " (DEG) (DEG)   (MRAD)   (MRAD) (DEG/DEG) (DEG/DEG)   (DB)"//)
      WRITE(7,2) TITLE,FINR(NG),DIACS,ZTOPIN,FREQ,RA,RR,DAPWL,IPOL,
     $ICASE,IOPT
      DO 18 I=1,N
   18 WRITE(7,7) I,CIN(I),ER(I),TD(I)
      WRITE(7,9)
    2 FORMAT(1H1,5X," RESULTS OF RADOME ANALYSIS"/
     1 18A4/" FINENESS RATIO=",F6.2,2X,
     2"DIAMETER=",F8.5," IN.  LENGTH=",F8.5," IN."/" FREQUENCY=",
```

```
3F7.3," GHZ "/
4" RA=",F8.5," IN.   FR=",F8.5," IN.   ANTENNA D=",F8.4,
5" WAVELENGTHS"/" IPCL=",I2,"   ICASE=",I2,"   IOPT=",I2//
6" LAYER   THICKNESS(IN.)    ER       TAND"/)
 DO 100 IPHI=1,NPHI
 PHIP=PHI(IPHI)
 PHIP=PHIP+180.
 PHIR=PHIR*RAD
 DO 100 ITHE=1,NTHE
 THETAL=THETA(ITHE)
 THETAR=180.-THETAL
 THETAR=THETAR*RAD
 CALL ORIENT(RA,THETAA,PHIA,FR,THETAR,PHIR,AGAM3A,ROTATE,TRANSL)
 IF (TABLE) GO TO 23
C   COMPUTE NORMALIZING FACTOR:
 KA(1)=0.
 KA(2)=0.
 KA(3)=1.
 CALL INCPW(KA,PWI,ICPT)
 TSUP=.TRUE.
 TABLE=.FALSE.
 CALL RECM(PWI,KA,NX,NY,KXMAX,KYMAX,FREQ,ROTATE,TRANSL,
$SUMX,SUMY,DELX,DELY,DAZX,DAZY,VR,TABLE,TSUP,RSQMAX)
 VAIRM=CABS(VR(1))
 TABLE=.TRUE.
23 IF (.NOT.SUPPRS) GO TO 24
 GO TO 350
24 CONTINUE
 DO 320 J=1,2
 ICUT=J
 ICOMP=IOPT
 KMAX=.995
 IF (KMAX.GT.KXMAX) KMAX=KXMAX
 TSUP=.TRUE.
 CALL RECPTN(SUMX,SUMY,DELX,DELY,DAZX,DAZY,NX,NY,ICUT,ICOMP,KMAX,
$NREC,VREC3,KXMAX,KYMAX,FREQ,ROTATE,TRANSL,TABLE,TSUP,RSQMAX)
 DO 325 M1=1,3
 ICHAN=MM
```

```
      IF ((ICUT.EQ.1).AND.(ICHAN.EQ.3)) GO TO 325
      IF ((ICUT.EQ.2).AND.(ICHAN.EQ.2)) GO TO 325
      DO 26 I=1,NREC
   26 VREC(I)=VREC3(I,ICHAN)
      IF (NREC.GT.NXE) GO TO 31
      CALL MAGFFT(VREC,NREC,XYFFT,NXE)
      DO 305 I=1,NXE
  305 MVREC(I)=CABS(XYFFT(I))**2
      GO TO 33
   31 DO 32 I=1,NREC
   32 MVREC(I)=CABS(VREC(I))**2
   33 NXX=MAX0(NXE,NREC)
      WRITE(6,306)
  306 FORMAT(/" MIN AND MAX VALUES OF REC""G PATTERN: "/)
      CALL NORMH(MVREC,NXX,1,.FALSE.)
      CALL DBPV(MVREC,NXX,1,1)
      IF (J.EQ.1) WRITE(6,308)
      IF (J.EQ.2) WRITE(6,309)
      DK=2.*KMAX/NXX
      DO 307 I=1,NXX,1
      IF (SUPPRS) GO TO307
      ANG=ASIN(-KMAX+(I-1)*DK)*180./PI
      CALL AMPHS(XYFFT(I),AMP,PHS)
      IF (NREC.GE.NXE) CALL AMPHS(VREC(I),AMP,PHS)
      IF (MOD(I,4).EQ.0) WRITE(6,310) ANG,MVREC(I),PHS
  307 MVREC(I)=1.0+MVREC(I)/40.
  308 FORMAT(/" REC""G PATTERN, EL CUT, EL COMP (DB): "/)
  309 FORMAT(/" REC""G PATTERN, AZ CUT,EL COMP (DB): "/)
  310 FORMAT(F9.1,5X,F8.3,3X,F6.1)
      IF (.NOT.GRAFRV) GO TO 320
      CALL CNPLTH(MVREC,NXX,KMAX,0.,0.)
      IF (J.EQ.1) CALL SYMBOL(.5,6.5,.140,43HFIGURE       RECVG POWER PA
     $TTERN-ELEV PLANE,0.,43)
      IF (J.EQ.2) CALL SYMBOL(.5,6.5,.140,41HFIGURE       RECVG POWER PA
     $TTERN-AZ PLANE,0.,41)
      CALL PLOT(8.5,0.,-3)
  325 CONTINUE
  320 CONTINUE
```

```
  350 CONTINUE
C COMPUTE BORESIGHT ERROR
  275 CONTINUE
      CALL RECBS(SUMX,SUMY,DELX,DELY,DAZX,DAZY,NX,NY,
     $ LMAX,NS,ICPT,VF,OMRAD,ROTATE,TRANSL,FREQ,KXMAX,KYMAX,
     $ TABLE,SINCS,KA,AZT,ELT,RSQMAX,VMAX,SMAX,SUPPRS)
      IF (ITHE.EQ.1) GO TO 300
      SLPAZ=(AZT-AZL)*DPMR/(THETAL-THL)
      SLPEL=(ELT-ELL)*DPMR/(THETAL-THL)
  300 AZL=AZT
      ELL=ELT
      THL=THETAL
      GAINM=20.*ALOG10(SMAX/VAIRM)
      WRITE(6,11) PHIP,THETAL,ELT,AZT,SLPEL,SLPAZ,GAINM
      WRITE(7,11) PHIP,THETAL,ELT,AZT,SLPEL,SLPAZ,GAINM
   11 FORMAT(1X,F5.1,F6.1,F8.2,F8.2,F9.4,F10.4,F7.1)
C  GRAF3D OPTION HAS BEEN REMOVED.
  100 CONTINUE
      WRITE(6,105) VAIRM
  105 FORMAT(//" RECEIVED SUM VOLTAGE WITHOUT RADOME=",E12.5//)
      IF (GRAF3D.OR.GRAFSA.OR.GRAFTR.OR.GRAFRV) CALL PLOT(0.,0.,999)
      STOP
      END
```

```
      BLOCK DATA
C
      COMMON/TRANSC/DIN(6),ER(6),TD(6),TZ,WALTOL,N,NN,D(6),ZB,TK
C
      DATA WALTOL,TK,TZ/0.,0.,0./
C
      END
```

Chapter 3

SUBROUTINE HACNF

3-1. Purpose: To compute near-field aperture distributions for two types of three-channel monopulse antennas: (1) circular aperture with uniform amplitude and phase distributions; (2) flat plate antenna with a programmed amplitude distribution and uniform phase. Four polarizations can be selected for the circular aperture. The flat plate antenna is vertically ($\hat{y}_A$) polarized only.

3-2. Usage: CALL HACNF (E, NX, NY, ICHAN, IPOL, IXY, DAPWL, DXWL, KXMAX, ICASE)

3-3. Arguments

E - Complex array of NX by NY elements which, on output, contains the values of the specified (IXY) rectangular component ($\hat{x}_A$ or $\hat{y}_A$) of the electric field distribution over the specified (ICASE) antenna aperture having the specified (IPOL) polarization for the specified (ICHAN) channel of a three-channel monopulse antenna.

NX,NY - Even integer number of points in a rectangular array at which the aperture distribution is computed in the x_A and y_A directions, respectively. The point I=NX/2 + 1, J=NY/2 + 1 corresponds to $x_A=0$, $y_A=0$.

[illegible] - Integer control variable with values 1, 2, or 3 which selects the sum, elevation difference, or azimuth difference channel, respectively.

[illegible] - Integer control variable which selects the antenna polarization as follows:

1 - Vertical (y_A) polarization

2 - Horizontal (x_A) "

3 - Right-hand circular "

4 - Left-hand circular "

IXY - Integer control variable having values 1 or 2 to select the x_A or y_A component of aperture electric field.

DAPWL - Diameter, in wavelengths, of the antenna aperture.

DXWL - Spacing, in wavelengths, between samples in aperture in x_A and y_A directions (output).

KMAX - Maximum value of normalized wavenumber corresponding to KMAX = 1./(2.*DXWL) (output).

ICASE - Integer control variable having values 1 or 2 to specify a circular aperture antenna with uniform amplitude and phase. If ICASE=3, a flat plate antenna having a programmed amplitude distribution (see Table 3-2) with vertical polarization is selected.

3-4. Comments and Method

a. The integers NX,NY must each be equal to each other and to an integer power of two; e.g., NX=NY=16. In addition, when ICASE=3 (flat plate antenna), NX and NY must equal 16.

b. The actual shape of the circular aperture, as approximated by a rectangular array of sample points, is shown in Figure 3-1 for the case of NX=NY=16. Row 1 and Column 1 of the array contain null elements. The elements inside and on the boundary of the aperture may contain non-zero values as shown in Table 3-1 for the various cases when ICHAN=1 (sum channel). Note that specification of D_{AP} in Figure 3-1 determines the sample spacings according to

$$\Delta x_A = \Delta y_A = \frac{D_{Ap} \cos\alpha}{(N_x - 2)} = \frac{D_{Ap} \cos\alpha}{(N_y - 2)} \tag{1}$$

where $\alpha = \mathrm{Tan}^{-1}(2/7)$.

The aperture distributions for three monopulse channels are formed by phasing the elements in the four quadrants of the aperture appropriately. The sum channel distribution is formed by assigning equal phases to all elements. The azimuth difference channel is formed by multiplying all elements in Quadrants II and III of the sum distribution by minus one and by zeroing all elements along $x_A=0$. For the elevation difference channel, Quadrants III and IV are negated, and all elements along the line $y_A=0$ are made zero for symmetry reasons.

The phasing chosen models a tracking antenna and provides outputs in two orthogonal channels from which the direction of arrival of a target return can be mathematically determined. Let $\hat{k}$ be a unit vector which points from the antenna origin toward the direction from whence the plane wave (target return) emanates; i.e.,

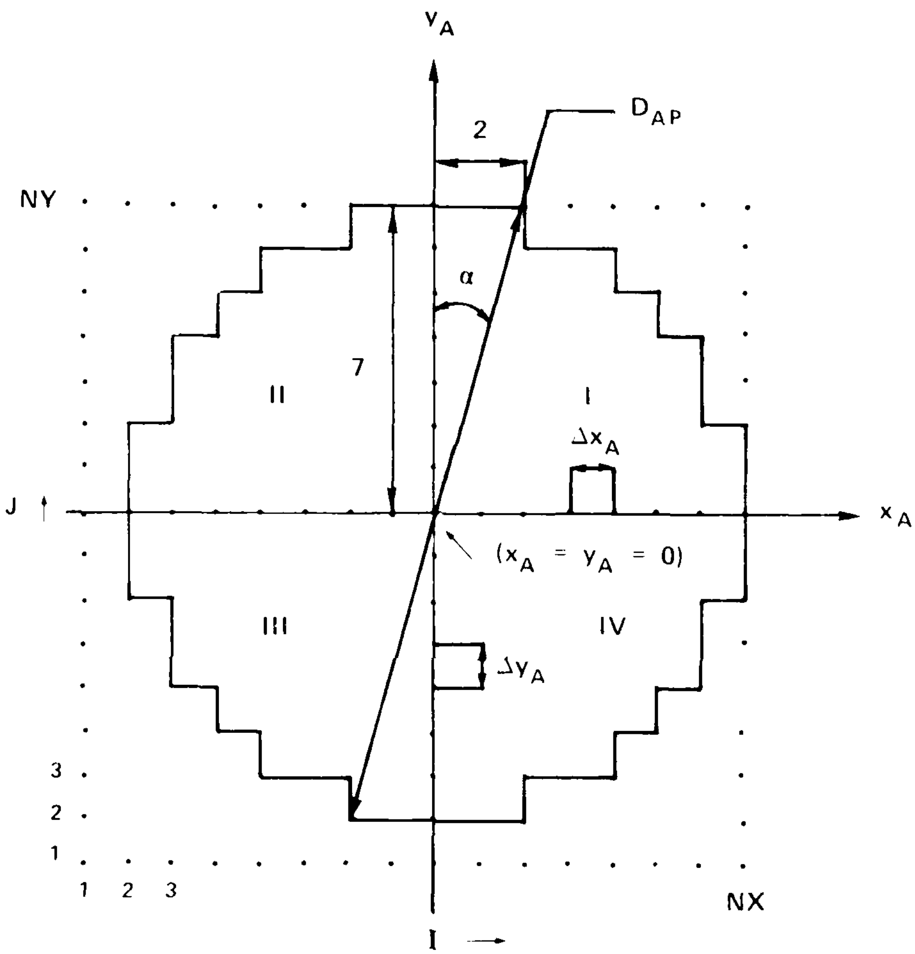

FIGURE 3–1. APPROXIMATION OF CIRCULAR APERTURE BY RECTANGULAR GRID OF SAMPLE POINTS.

Table 3-1. Values of Non-Zero Elements in Circular Aperture (ICHAN=1, ICASE=1 or 2)

IPOL	IXY	Value	Polarization Type
1	1	(0 + j0)	Vertical
1	2	(1 + j0)	"
2	1	(1 + j0)	Horizontal
2	2	(0 + j0)	"
3	1	(0 + j1)	RHC
3	2	(1 + j0)	"
4	1	(0 - j1)	LHC
4	2	(1 + j0)	"

$$\hat{k} = \hat{x}_A k_x + \hat{y}_A k_y + \hat{z}_A k_z \qquad (2)$$

Define the tracking functions for this plane wave as

$$f_i(k_x, k_y) = \frac{\Delta_i(k_x, k_y)}{\Sigma(k_x, k_y)} \qquad (3)$$

where Δ_i represents the output of the elevation (ϵ) or azimuth (α) difference channel and Σ represents the sum channel output. Then for small $k_x > 0$, the phase of f_α is $+ \pi/2$; for small $k_x < 0$, the phase of f_α is $-\pi/2$. Similarly, for small $k_y > 0$, arg $(f_\epsilon) = \pi/2$; for small $k_y < 0$, arg $(f_\epsilon) = -\pi/2$. Hence, the change in phase by π in either channel represents the boresight direction of the antenna, and tracking is done using the imaginary parts of the tracking functions rather than their real parts.

c. The shape and sampling grid used to model the flat plate antenna are shown in Figure 3-2. In Subroutine [illegible]CNF, the integers NX and NY must both equal 16, and only linear polarization ($\hat{y}_A$) is applicable to the flat plate antenna (ICASE=3). The phasing of the four quadrants is done as described above to model the three monopulse channels so that tracking can be simulated. Note that specification of D_{AP} determines the sample spacing according to

$$\Delta x_A = \Delta y_A = \frac{D_{AP} \cos\alpha}{\left(\frac{N_x}{2} - 2\right)} \qquad (4)$$

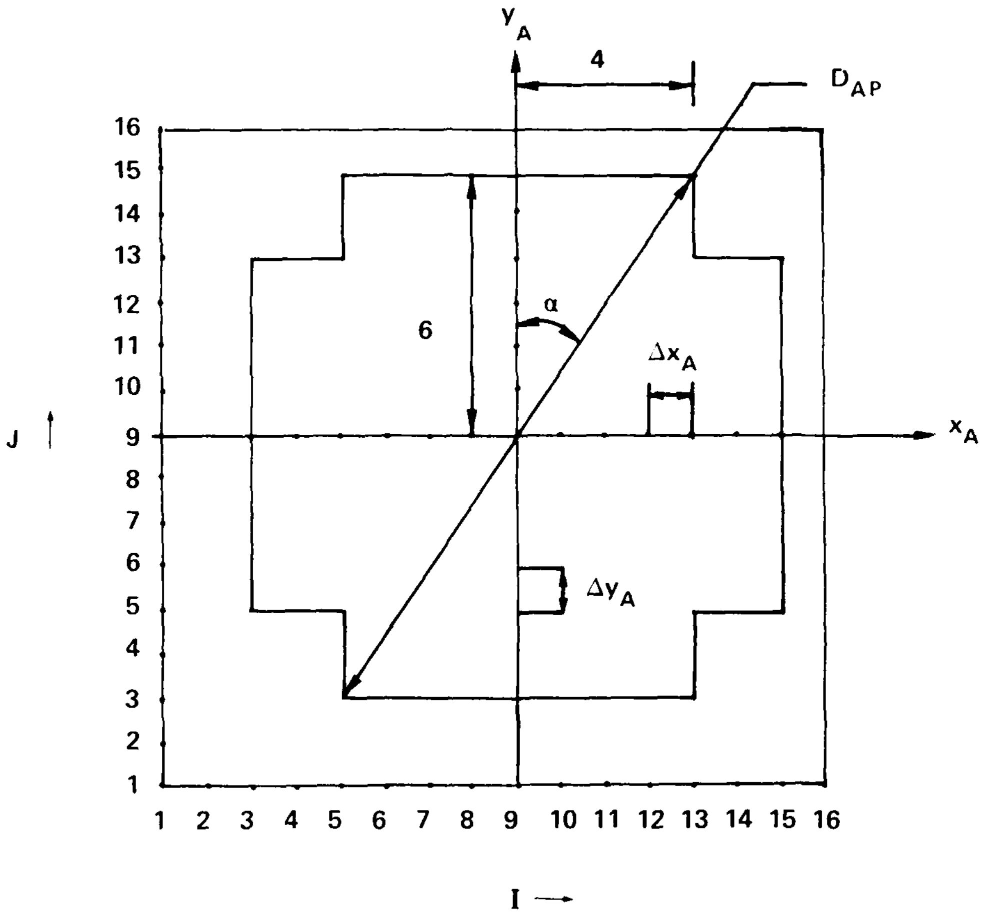

FIGURE 3–2. GEOMETRY OF FLAT PLATE ANTENNA.

where $\alpha = \mathrm{Tan}^{-1}(4/6)$.

The phase of each sample point in Figure 3-2 for the sum channel is made equal, but the amplitudes are tapered in the x_A and y_A directions as shown in Table 3-2. The amplitude distribution is separable and symmetrical so that

$$E_{yA}(x_A,y_A) = a(x_A)b(y_A) \qquad E_{yA}(-x_A,y_A) = E_{yA}(x_A,-y_A) \tag{5}$$

It is noted that samples 10, 12, 14, and 16 are actually specified in the program, and samples 9, 11, 13, and 15 are obtained from them by averaging.

3-5. Program Flow

Line 16: Assign complex values to CFAC to use in generating vertical, horizontal, RHC, and LHC polarization according to IPOL.

Lines 17-19: Compute the angle α and the upper bound R_{max} of the radius of the circular aperture.

Lines 20-21: Ensure that IPOL has correct values of 1, 2, 3, or 4.

Line 22: If NX≠NY, write error message and stop the program.

Line 23: Ensure that IXY = 1 or 2.

Line 24: If NX and NY are not even, stop the program.

Line 25: Test value of ICASE: if ICASE=3 generate fields of flat plate antenna (Lines 47-83); otherwise, generate fields of circular aperture (Lines 26-43).

Lines 26-41: Assign complex field value to each sample point $(x_A,y_A,0)$ in the aperture according to the values shown in Table 3-1. If $\sqrt{x_A^2 + y_A^2} > R_{max}$, make the

Table 3-2. Symmetrical Amplitude Distribution for Flat Plate Antenna

Sample No.	x_A	Amplitude	y_A	Amplitude
9	0	1.0280	0	1.0280
10	Δx	1.0280	Δy	1.0280
11	$2\Delta x$	.9120	$2\Delta y$	.9170
12	$3\Delta x$	.7959	$3\Delta y$	.8060
13	$4\Delta x$	.6077	$4\Delta y$	.6155
14	$5\Delta x$	.4194	$5\Delta y$	.4250
15	$6\Delta x$	.2097	$6\Delta y$	.2125
16	$7\Delta x$	0.0	$7\Delta y$	0.0

field value zero (Line 40). Multiply the non-zero elements by CFAC(IPOL) to generate the correct polarization (Line 38).

Lines 42-43: Compute sample spacing $\Delta x_A/\lambda$ and go to statement 60.

Lines 44-46: Error message and STOP.

Line 47-48: Flat plate antenna-- if NX≠16, write error message and STOP (Lines 109-111).

Line 49: Compute sample spacing $\Delta x_A/\lambda$.

Line 50: Ensure NX=NY

Lines 51-54: Zero all elements in the aperture. If IXY=1 (x_A-component), go to statement 60.

Lines 55-62: Assign tapered amplitude values to eight "even" elements in Quadrant III.

Lines 63-71: Compute amplitude values for the "odd" elements in Quadrant III.

Lines 72-75: Compute amplitude values for elements 3-9 along y_A=0 line and along x_A=0 line.

Lines 76-79: Generate symmetrical amplitude values in Quadrant IV.

Lines 80-83: Generate symmetrical amplitude values in Quadrants I and II.

Line 84: Compute k_{xmax}.

Lines 85-89: Test to determine if the sum channel data generated should be phased to produce the aperture distribution for a specified difference channel (ICHAN).

Lines 90-98: Form aperture distribution for difference elevation channel by zeroing all elements along y_A=0 and negating all elements for y_A<0. RETURN.

Lines 99-107: Form aperture distribution for difference azimuth channel by zeroing all elements along $x_A=0$ and negating all elements for $x_A<0$. RETURN.

Lines 108-112: Error message for ICASE=3 and $NX \neq 16$. Comment of DAPWL=5.047 applies to the test described in Chapter 2. END

3-6. Test Case: See discussion in Chapter 2.

3-7. References

1. D. R. Rhodes, Introduction to Monopulse, McGraw Hill, New York, 1959.

3-8. Program Listing: See following pages.

```
      SUBROUTINE HACNF(E,NX,NY,ICHAN,IPOL,IXY,DAPWL,DXWL,KXMAX,ICASE)
C SUBR HACNF COMPUTES ELECTRIC FIELDCOMPONENTS OVER A CIRCULAR APERTURE
C OF RADIUS RMAX=(NX/2-1)/COS(ATAN(2/7)) AND RETURNS SAME IN E(NX,NY).
C NX MUST EQUAL NY AND MUST BE EVEN.
C  ICHAN=1 FOR SUM CHANNEL    IPOL=1 FOR VERT-Y POL.    IXY=1 FOR X-COMP.
C       =2 FOR ELEV DIFF          =2 FOR HORIZ-X POL        =2 FOR Y-COMP.
C       =3 FOR AZ DIFF            =3 FOR RHC POL
C       =                         =4 FOR LHC POL
C DAPWL=DIAMETER OF APERTURE IN WAVELENGTHS (INPUT)
C  DXWL=SAMPLE SPACING IN APERTURE  (OUTPUT)
C KXMAX=MAXIMUM WAVENUMBER (OUTPUT)
C  ICASE=1 OR 2 FOR UNIFORM, CIRCULAR APERTURE (CASE I AND II)
C       =3 FOR FLAT-PLATE ANTENNA, VERTICAL POL (CASE III).
      COMPLEX E(NX,NY),CFAC(4)
      REAL KXMAX
      DATA CFAC/(1.,0.),(1.,0.),(0.,+1.),(0.,-1.)/
      ANG=ATAN(2./7.)
      IF (ICASE.EQ.3) ANG=ATAN(4./6.)
      RMAX=(NX/2-1)/COS(ANG)+.001
      IF (IPOL.GT.4) IPOL=4
      IF (IPOL.LT.1) IPOL=1
      IF (NX.NE.NY) GO TO 15
      IF ((IXY.LT.1).OR.(IXY.GT.2)) IXY=2
      IF (MOD(NX,2).NE.0) GO TO 15
      IF (ICASE.EQ.3) GO TO 25
      DO 10 I=1,NX
      X=FLOAT(-(NX/2)+I-1)
      DO 10 J=1,NY
      Y=FLOAT(-(NY/2)+J-1)
      R=SQRT(X**2+Y**2)
      IF (R.GT.RMAX) GO TO 9
      IF ((IPOL.EQ.1).AND.(IXY.EQ.1)) GO TO 9
      IF ((IPOL.EQ.2).AND.(IXY.EQ.2)) GO TO 9
C  IF RHC, EY=(1,0), EX=(0,1)  I.E., EX LEADS EY BY 90 DEG.
C  IF LHC,EY=(1,0), EX=(0,-1)  I.E., EX LAGS EY BY 90 DEG.
      E(I,J)=(1.,0.)
      IF ((IPOL.LT.3).OR.(IXY.EQ.2)) GO TO 10
      E(I,J)=E(I,J)*CFAC(IPOL)
```

```
      GO TO 10
    9 E(I,J)=(0.,0.)
   10 CONTINUE
      DXWL=(DAPWL/2.)*COS(ANG)/(NX/2-1)
      GO TO 60
   15 WRITE(6,20)
   20 FORMAT(//" NX.NE.NY OR NX NOT EVENIN SUBR HAGNF"//)
      STOP
C THE FOLLOWING IS FOR CASE III (ICASE=2):
   25 IF (NX.NE.16) GO TO 90
      DXWL=(DAPWL/2.)*COS(ANG)/(NX/2-2)
      NY=NX
      DO 26 I=1,NX
      DO 26 J=1,NY
   26 E(I,J)=(0.,0.)
      IF (IXY.EQ.1) GO TO 60
      E(6,4)=(.2824,0.)
      E(8,4)=(.4250,0.)
      E(4,6)=(.2888,0.)
      E(6,6)=(.5218,0.)
      E(8,6)=(.8060,0.)
      E(4,8)=(.4194,0.)
      E(6,8)=(.7959,0.)
      E(8,8)=(1.028,0.)
      DO 30 J=4,8,2
      DO 30 I=3,8,1
      IF ((MOD(J,2).EQ.0).AND.(MOD(I,2).EQ.0)) GOTO 30
      E(I,J)=(E(I-1,J)+E(I+1,J))/2.
   30 CONTINUE
      DO 35 I=3,8,1
      DO 35 J=3,8,2
      E(I,J)=(E(I,J-1)+E(I,J+1))/2.
   35 CONTINUE
      DO 40 I=3,9
   40 E(I,J)=E(I,8)
      DO 45 J=3,9
   45 E(9,J)=E(8,J)
      DO 50 J=3,9
```

```
      DO 50 I=1,6
      E(9+I,J)=E(9-I,J)
   50 CONTINUE
      DO 55 I=3,15
      DO 55 J=1,6
      E(I,9+J)=E(I,9-J)
   55 CONTINUE
   60 KXMAX=1./(2.*DXWL)
      IF (ICHAN.EQ.1) RETURN
      IF ((IXY.EQ.1).AND.(ICASE.EQ.3)) RETURN
      IF ((IXY.EQ.1).AND.(IPOL.EQ.1)) RETURN
      IF ((IXY.EQ.2).AND.(IPOL.EQ.2)) RETURN
      IF (ICHAN.EQ.3) GO TO 75
C   LOAD ELEVATION DIFFERENCE CHANNEL*
      J=NY/2+1
      DO 65 I=1,NX
   65 E(I,J)=(0.,0.)
      JMAX=NY/2
      DO 70 J=1,JMAX
      DO 70 I=1,NX
   70 E(I,J)=-E(I,J)
      RETURN
C   LOAD AZIMUTH DIFFERENCE CHANNEL*
   75 I=NX/2+1
      DO 80 J=1,NY
   80 E(I,J)=(0.,0.)
      IMAX=NX/2
      DO 85 I=1,IMAX
      DO 85 J=1,NY
   85 E(I,J)=-E(I,J)
      RETURN
C   DAPWL=5.047 FOR CASE III
   90 WRITE(6,95)
   95 FORMAT(//"***ERROR EXIT! NY NOT EQUAL TO 16 IN SUBR HACNF***"//)
      STOP
      END
```

Chapter 4

SUBROUTINE ORIENT

4-1. Purpose: To compute the rotational matrix of direction cosines ROTATE and the translational matrix TRANSL required to carry out coordinate and vector transformations between antenna coordinate system (x_A, y_A, z_A) and radome coordinate system (x_R, y_R, z_R).

4-2. Usage: CALL ORIENT (RA, THETA, PHIA, RR, THETAR, PHIR, AGAMMA,

ROTATE, TRANSL)

4-3. Arguments

RA, THETA, PHIA - Spherical coordinates (cm, radians) of the origin of the antenna coordinate system with respect to the reference coordinate system (x,y,z) as indicated in Figure 4-1. Note that the origin of the reference system coincides with the gimbal point, which is located on the axis of symmetry z_R of the radome.

RR, THETAR, PHIR - Spherical coordinates (cm, radians) of the origin of the radome coordinate system with respect to the reference system.

AGAMMA - Angle (radians) between the z_A and z axes.

ROTATE - Real array of 3 x 3 elements which contains on output the matrix of direction cosines $[R_i]$ explained below.

TRANSL - Real array of three elements which contains on output the translation matrix T_i as explained below.

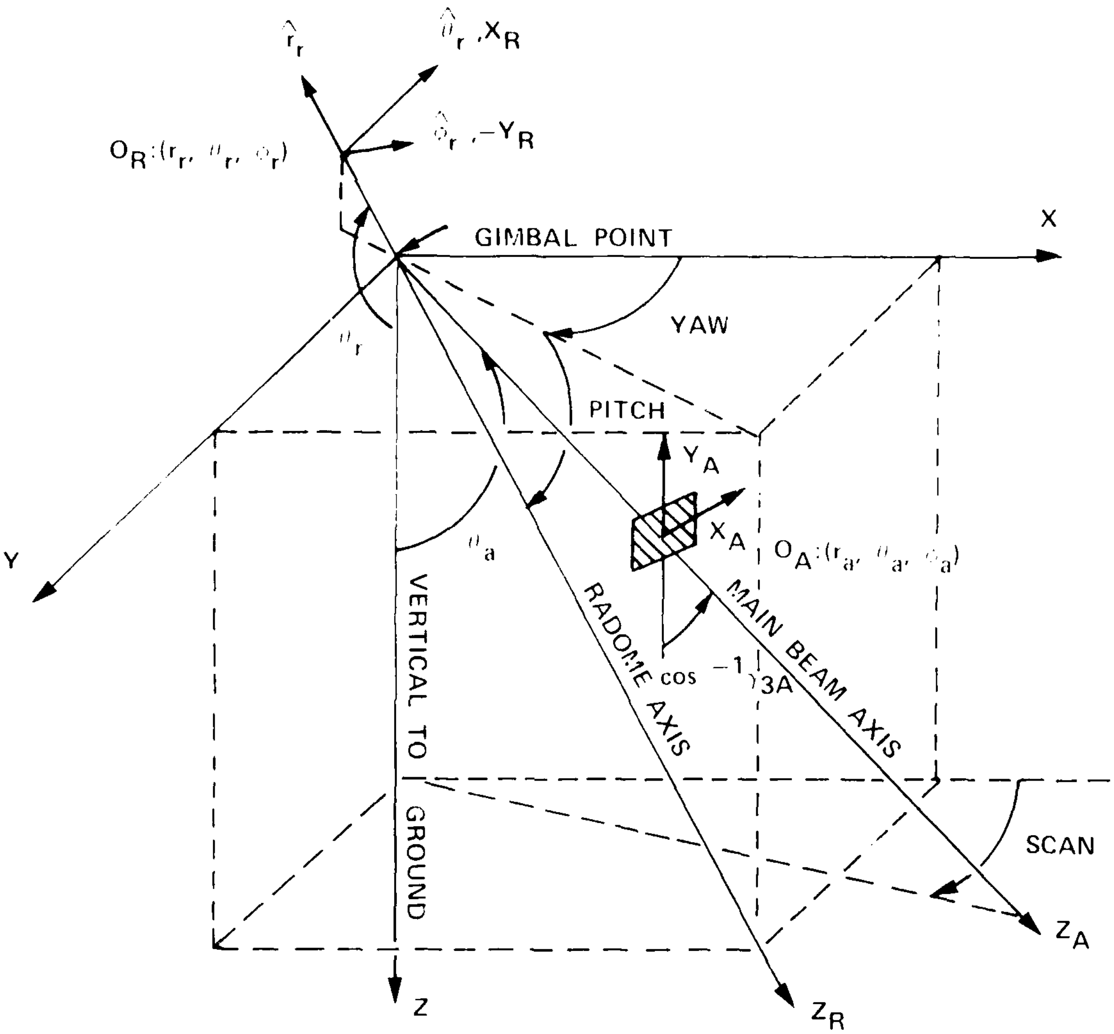

Figure 4–1. Coordinate Systems Used in Radome Analysis.

Reference System: (X, Y, Z)

Antenna System: (X_A, Y_A, Z_A)

Radome System: (X_R, Y_R, Z_R)

4-4. Comments and Method

a. The coordinate systems of Figure 2-2 are obtained from those shown in Figure 4-1 by setting THETAA = 0, PHIA = $\pi/2$, and AGAM3A = 0. When this is done, the z and z_A axes coincide, the x and x_A axes are parallel, and the y and y_A axes are parallel. The angles ϕ_p and θ_L in Figure 2-2 are related to θ_r and ϕ_r as

$$\phi_r = \phi_p + \pi \tag{1}$$

$$\theta_r = \pi - \theta_L \tag{2}$$

The unit vectors $\hat{x}_R$, $\hat{y}_R$, $\hat{z}_R$ were chosen to coincide with the spherical coordinate unit vectors $\hat{r}_r$, $\hat{\theta}_r$, $\hat{\phi}_r$ associated with the point O_R: (r_r, θ_r, ϕ_r); hence, $\hat{x}_R$ always lies in the plane of scan as indicated in Figure 4-2. This observation is important if the properties of the radome wall are not symmetric with respect to rotation about the z_R-axis, such as in the case of circumferential variations in wall thickness, nonuniform heating, etc.

b. The details of the coordinate system transformations are described in Reference 1 and summarized below. The transformation of the point P in antenna coordinates (x_A, y_A, z_A) to radome coordinates (x_R, y_R, z_R) is given by

$$\begin{bmatrix} x_R \\ y_R \\ z_R \end{bmatrix} = \begin{bmatrix} R_{ij} \end{bmatrix} \begin{bmatrix} x_A \\ y_A \\ z_A \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} \tag{3}$$

The transformation of a vector

$$\underline{F} = \hat{x}_A F_{xA} + \hat{y}_A F_{yA} + \hat{z}_A F_{zA} = \hat{x}_R F_{xR} + \hat{y}_R F_{yR} + \hat{z}_R F_{zR} \tag{4}$$

is given by

$$\begin{bmatrix} F_{xR} \\ F_{yR} \\ F_{zR} \end{bmatrix} = \begin{bmatrix} R_{ij} \end{bmatrix} \begin{bmatrix} F_{xA} \\ F_{yA} \\ F_{zA} \end{bmatrix} \tag{5}$$

In the above, $[R_{ij}]$ is the matrix of direction cosines which describes the rotation of the radome coordinate system with respect to the antenna system; $[T_i]$ describes the translation of the radome origin O_r with respect to the origin O_a of the antenna system. In fact, setting ($x_A = 0$, $y_A = 0$, $z_A = 0$) in Equation (3) shows that (T_x, T_y, T_z) represents the location, in radome coordinates, of the antenna origin.

The matrix $[R_{ij}]$ can be expanded and written explicitly as

$$\begin{bmatrix} R_{ij} \end{bmatrix} = \begin{bmatrix} \cos \alpha_1 & \cos \beta_1 & \cos \gamma_1 \\ \cos \alpha_2 & \cos \beta_2 & \cos \gamma_2 \\ \cos \alpha_3 & \cos \beta_3 & \cos \gamma_3 \end{bmatrix} \tag{6}$$

where

$$\alpha_1 = \text{angle between } x_A \text{ and } x_R = \angle\, x_A,\ x_R \tag{7a}$$

$$\alpha_2 = \angle\, x_A,\ y_R \tag{7b}$$

$$\alpha_3 = \angle\, x_A,\ z_R \tag{7c}$$

$$\beta_1 = \angle\, y_A,\ x_R \tag{7d}$$

$$\beta_2 = \angle\, y_A,\ y_R \tag{7e}$$

$$\beta_3 = \angle\, y_A,\ z_R \tag{7f}$$

$$\gamma_1 = \angle\, z_A,\ x_R \tag{7g}$$

$$\gamma_2 = \angle\, z_A,\ y_R \tag{7h}$$

$$\gamma_3 = \angle\, z_A,\ z_R \tag{7i}$$

The inverse transformations are given by

$$\begin{bmatrix} x_A \\ y_A \\ z_A \end{bmatrix} = \begin{bmatrix} R_{ij} \end{bmatrix}^T \left\{ \begin{bmatrix} x_R \\ y_R \\ z_R \end{bmatrix} - \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} \right\} \tag{8}$$

$$\begin{bmatrix} F_{xA} \\ F_{yA} \\ F_{zA} \end{bmatrix} = \begin{bmatrix} R_{ij} \end{bmatrix}^T \begin{bmatrix} F_{xR} \\ F_{yR} \\ F_{zR} \end{bmatrix} \tag{9}$$

where $[R_{ij}]^T$ denotes the transpose of $[R_{ij}]$; i.e., $[R_{ij}]^T = [R_{ji}]$ since rows and columns are interchanged. Also, since $[R_{ij}]$ is a unitary matrix, its inverse is equal to its transpose.

To facilitate the specification of a particular antenna/radome orientation, the reference coordinate system (x, y, z) is used. Transformations from the reference system to the antenna system are described by

$$\begin{bmatrix} x_A \\ y_A \\ z_A \end{bmatrix} = \begin{bmatrix} \gamma_{ij} \end{bmatrix} \begin{bmatrix} x - r_a \sin\theta_a \cos\phi_a \\ y - r_a \sin\theta_a \sin\phi_a \\ z - r_a \cos\theta_a \end{bmatrix} \tag{10}$$

while transformations from reference system to radome system are described by

$$\begin{bmatrix} x_R \\ y_R \\ z_R \end{bmatrix} = \begin{bmatrix} \rho_{ij} \end{bmatrix} \begin{bmatrix} x - r_r \sin\theta_r \cos\phi_r \\ y - r_r \sin\theta_r \sin\phi_r \\ z - r_r \cos\theta_r \end{bmatrix} \tag{11}$$

where $[\gamma_{ij}]$ and $[\rho_{ij}]$ represent the rotations of the two systems with respect to the reference system. When these two separate transformations

are combined, there results

$$\begin{bmatrix} R_{ij} \end{bmatrix} = \begin{bmatrix} \rho_{ij} \end{bmatrix} \begin{bmatrix} \gamma_{ij} \end{bmatrix}^T \tag{12}$$

$$\begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = \begin{bmatrix} \rho_{ij} \end{bmatrix} \begin{bmatrix} X_a - X_r \\ Y_a - Y_r \\ Z_a - Z_r \end{bmatrix} \tag{13}$$

where X_a, Y_a, etc., are defined in Equations (10) and (11); e.g., $X_a = r_a \sin\theta_a \cos\phi_a$.

4-5. Program Flow: See listing below and compare directly to method described above. Note that in GAM(I,J), the index I represents the row number in $[\gamma_{ij}]$, and index J represents the column number.

4-6. Test Case: See discussion in Chapter 2.

4-7. Reference

1. E. P. Joy and G. K. Huddleston, "Radome Effects on the Performance of Ground Mapping Radar," Technical Report, Contract DAAH01-72-C-05[illegible], U. S. Army Missile Command, March 1973.

4-8. Program Listing: See following pages.

```
      SUBROUTINE ORIENT(RA,THETAA,PHIA,RR,THETAR,PHIR,AGAM3A,
     -   ROTATE,TRANSL)
      DIMENSION           ROTATE(3,3),          TRANSL(3)
      REAL    GAM(3,3),RHO(3,3)
      DIMENSION T(3)
      PSI=THETAA-AGAM3A
      GAM(1,1)=SIN(PHIA)
      GAM(1,2)=-COS(PHIA)
      GAM(1,3)=0.
      GAM(2,1)=SIN(PSI)*SIN(THETAA)*COS(PHIA)+COS(PSI)*COS(THETAA)
     -   *COS(PHIA)
      GAM(2,2)=SIN(PSI)*SIN(THETAA)*SIN(PHIA)+COS(PSI)*COS(THETAA)
     -   *SIN(PHIA)
      GAM(2,3)=SIN(PSI)*COS(THETAA)-COS(PSI)*SIN(THETAA)
      GAM(3,1)=COS(PSI)*SIN(THETAA)*COS(PHIA)-SIN(PSI)*COS(THETAA)
     -   *COS(PHIA)
      GAM(3,2)=COS(PSI)*SIN(THETAA)*SIN(PHIA)-SIN(PSI)*COS(THETAA)
     -   *SIN(PHIA)
      GAM(3,3)=COS(AGAM3A)
      RHO(1,1)=COS(THETAR)*COS(PHIR)
      RHO(1,2)=COS(THETAR)*SIN(PHIR)
      RHO(1,3)=-SIN(THETAR)
      RHO(2,1)=SIN(PHIR)
      RHO(2,2)=-COS(PHIR)
      RHO(2,3)=0.
      RHO(3,1)=-SIN(THETAR)*COS(PHIR)
      RHO(3,2)=-SIN(THETAR)*SIN(PHIR)
      RHO(3,3)=-COS(THETAR)
      XA=RA*SIN(THETAA)*COS(PHIA)
      YA=RA*SIN(THETAA)*SIN(PHIA)
      ZA=RA*COS(THETAA)
      XR=RR*SIN(THETAR)*COS(PHIR)
      YR=RR*SIN(THETAR)*SIN(PHIR)
      ZR=RR*COS(THETAR)
C          COMPUTE THE ROTATE ARRAY BY MULTIPLYING THE RHO ARRAY
C              AND THE TRANSPOSE OF THE GAM ARRAY.
      DO 2 I=1,3
      DO 2 J=1,3
```

```
      ROTATE(I,J)=0.
      DO 2 K=1,3
      ROTATE(I,J)=ROTATE(I,J)+RHO(I,K)*GAM(J,K)
    2 CONTINUE
C         COMPUTE TRANSL ARRAY
      T(1)=XA-XB
      T(2)=YA-YB
      T(3)=ZA-ZB
      DO 10 I=1,3
      TRANSL(I)=0.0
      DO 10 J=1,3
      TRANSL(I)=TRANSL(I)+RHO(I,J)*T(J)
   10 CONTINUE
      RETURN
      END
```

Chapter 5

SUBROUTINE POINT

5-1. Purpose: To transform a point P in antenna coordinates (x_A, y_A, z_A) to radome coordinates (x_R, y_R, z_R), and vice versa.

5-2. Usage: CALL POINT (P, PT, ATOR, T, PO)

5-3. Arguments

P	-	Real (input) array of three elements representing the Cartesian coordinates (cm) of the point to be transformed; e.g., P(1) = x_A, P(2) = y_A, P(3) = z_A.
PT	-	Real (output) array of three elements representing the Cartesian coordinates (cm) in the other coordinate system; e.g., PT(1) = x_R, etc.
ATOR	-	Logical input variable which controls direction of transformation: ATOR =.TRUE. for antenna-to-radome (see Equation (4-3)); ATOR = .FALSE. for radome-to-antenna coordinate transformation (Equation (4-8)).
T	-	Real (input) array of 3 x 3 elements representing the ROTATE array computed by Subroutine ORIENT.
PO	-	Real (input) array of three elements representing the TRANSL array computed by Subroutine ORIENT.

5-4. Comments and Method

a. Subroutines required: Subroutine ORIENT must be called prior to the first call to POINT so that T and PO are available.

b. For method, see Subroutine ORIENT in Chapter 4.

c. [illegible] listing below directly to Equations (4-[illegible]) and (4-8).

5-6. Test Case: See Chapter 2.

5-7. Reference: See Chapter 4.

5-8. Program Listing: See following page.

```
      SUBROUTINE POINT(P,PT,ATOR,T,PO)
C
C     THIS SUBROUTINE TRANSFORMS A POINT P GIVEN IN ONE COORDINATE SYSTEM
C     TO THE SAME POINT GIVEN IN ANOTHER COORDINATE SYSTEM, PT.
C     THE TRANSFORMATION IS ACCOMPLISHED USING THE TRANSFORM MATRIX T
C     THE LOGICAL VARIABLE ATOR DIRECTS WHICH TRANSFORM IS TO BE MADE
C     IF ATOR IS TRUE THE TRANSFORM IS FROM ANTENNA COORDINATES TO
C     RADOME COORDINATES.  IF ATOR IS FALSE THE OPPOSITE TRANSFORM IS MADE
C     T IS THE MATRIX OF DIRECTION COSINES WHICH DESCRIBE THE ROTATION
C     OF THE RADOME COORDINATE SYSTEM WITH RESPECT TO THE ANTENNA
C     COORDINATE SYSTEM, PO IS THE ORIGIN OF THE RADOME COORDINATE
C     SYSTEM IN ANTENNA COORDINATES
C
      REAL P(3),PT(3),T(3,3),PS(3),PO(3)
      LOGICAL ATOR
      IF(ATOR) GO TO 1
C
C     CONVERSION FROM RADOME TO ANTENNA COORDINATES
C
      PS(1)=P(1)-PO(1)
      PS(2)=P(2)-PO(2)
      PS(3)=P(3)-PO(3)
      PT(1)=T(1,1)*PS(1)+T(2,1)*PS(2)+T(3,1)*PS(3)
      PT(2)=T(1,2)*PS(1)+T(2,2)*PS(2)+T(3,2)*PS(3)
      PT(3)=T(1,3)*PS(1)+T(2,3)*PS(2)+T(3,3)*PS(3)
      RETURN
    1 CONTINUE
C
C     CONVERSION FROM ANTENNA TO RADOME COORDINATES
C
      PT(1)=T(1,1)*P(1)+T(1,2)*P(2)+T(1,3)*P(3)+PO(1)
      PT(2)=T(2,1)*P(1)+T(2,2)*P(2)+T(2,3)*P(3)+PO(2)
      PT(3)=T(3,1)*P(1)+T(3,2)*P(2)+T(3,3)*P(3)+PO(3)
      RETURN
      END
```

Chapter 6

SUBROUTINE VECTOR

6-1. Purpose: To transform a vector $\underline{F}$ in antenna coordinates to radome coordinates, and vice versa.

6-2. Usage: CALL VECTOR (V, VT, ATOR, T)

6-3. Arguments

V - Real (input) array of three elements representing the rectangular components of the vector to be transformed; e.g., V(1) = F_{xA}, V(2) = F_{yA}, V(3) = F_{zA}.

VT - Real (output) array of three elements representing the rectangular components of the vector in the other coordinate system; e.g., VT(1) = F_{xR}, etc.

ATOR - Logical input variable which controls the direction of the transformation: ATOR = .TRUE. for antenna-to-radome (Equation (4-[illegible])); ATOR = .FALSE. for radome-to-antenna (Equation (4-[illegible])).

T - Matrix ROTATE described in Chapter 4 as computed by Subroutine ORIENT.

6-4. Comments and Method

a. Subroutines required: Subroutine ORIENT must be called prior to the first call to VECTOR so that T will be available.

b. For method, see Subroutine ORIENT in Chapter 4.

6-5. Program Flow: Compare listing below directly to Equations (4-[illegible]) and (4-[illegible]).

6-6. Test Case: See Chapter 2.

6-7. References: See Chapter 4.

6-8. Program Listing: See following page.

```
      SUBROUTINE VECTOR(V,VT,ATOR,T)
C
C     THIS SUBROUTINE TRANSFORMS A VECTOR V GIVEN IN ONE COORDINATE SYSTEM
C     TO THE SAME VECTOR GIVEN IN ANOTHER COORDINATE SYSTEM, VT.
C     THE TRANSFORMATION IS ACCOMPLISHED USING THE TRANSFORM MATRIX T
C     THE LOGICAL VARIABLE ATOR DIRECTS WHICH TRANSFORM IS TO BE MADE
C     IF ATOR IS TRUE THE TRANSFORM IS FROM ANTENNA COORDINATES TO
C     RADOME COORDINATES.  IF ATOR IS FALSE THE OPPOSITE TRANSFORM IS MADE
C     T IS THE MATRIX OF DIRECTION COSINES WHICH DESCRIBE THE ROTATION
C     OF THE RADOME COORDINATE SYSTEM WITH RESPECT TO THE ANTENNA
C     COORDINATE SYSTEM.
C
      REAL V(3),VT(3),T(3,3)
      LOGICAL ATOR
      IF(ATOR) GO TO 1
C
C     CONVERSION FROM RADOME TO ANTENNA COORDINATES
C
      VT(1)=T(1,1)*V(1)+T(2,1)*V(2)+T(3,1)*V(3)
      VT(2)=T(1,2)*V(1)+T(2,2)*V(2)+T(3,2)*V(3)
      VT(3)=T(1,3)*V(1)+T(2,3)*V(2)+T(3,3)*V(3)
      RETURN
    1 CONTINUE
C
C      CONVERSION FROM ANTENNA TO RADOME COORDINATES
C
      VT(1)=T(1,1)*V(1)+T(1,2)*V(2)+T(1,3)*V(3)
      VT(2)=T(2,1)*V(1)+T(2,2)*V(2)+T(2,3)*V(3)
      VT(3)=T(3,1)*V(1)+T(3,2)*V(2)+T(3,3)*V(3)
      RETURN
      END
```

Chapter 7

SUBROUTINE INCPW

7-1. Purpose: To compute the rectangular vector components of the electric field of a plane electromagnetic wave propagating in the direction $-\hat{k}_A$ and incident on the $z_A=0$ plane of the antenna coordinate system, where $(x_A=0,\ y_A=0,\ z_A=0)$ is used as the phase origin.

7-2. Usage: CALL INCPW (KA, EI, IOPT)

7-3. Arguments

KA - Real input array of three elements containing the direction cosines of the direction $\hat{k}_A$ from whence the plane wave emanates; e.g., $KA(1) = k_{xA}$, $KA(2) = k_{yA}$, $KA(3) = k_{zA}$ where $\hat{k}_A = \hat{x}_A\, k_{xA} + \hat{y}_A\, k_{yA} + \hat{z}\, k_{zA}$.

EI - [illegible] output array of three elements containing th[illegible] rectangular components of the electric field E, [illegible] normalized such that [illegible] = 1.

[illegible] which [illegible] on [illegible] (see [illegible])

IOPT [illegible]

IOPT [illegible]

IOPT [illegible] right hand [illegible]

[illegible]

7-4. [illegible]

[illegible] is [illegible] th[illegible]

[illegible] of [illegible]

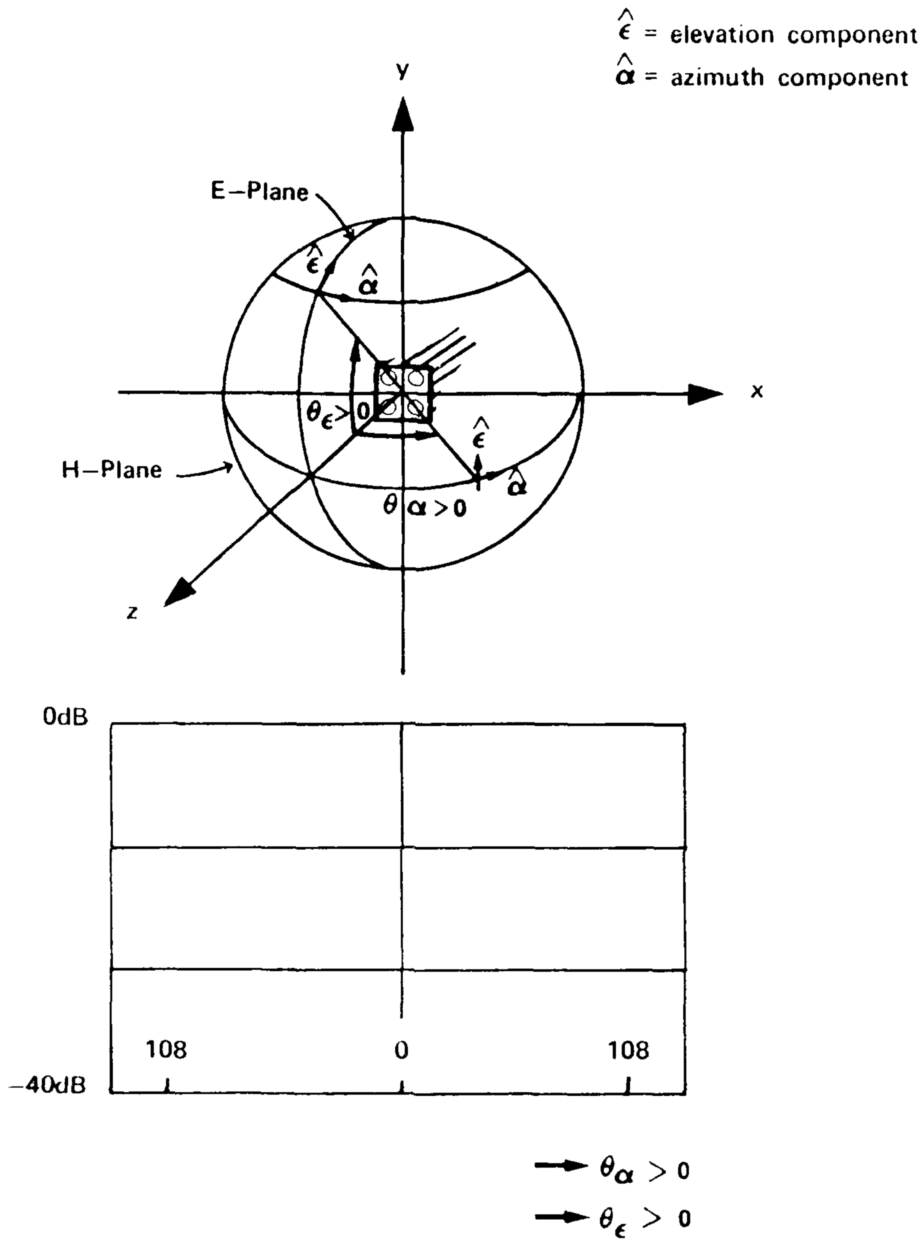

Figure - .Coordinate System for Far Field Patterns

will see the tip of the $\underline{E}$ vector trace out a circle in a plane of equal phase, with the direction of rotation being clockwise for right-hand circular and counterclockwise for left-hand circular.

b. Spherical to rectangular coordinate transformations [1] are used to define the rectangular vector components E_x, E_y, E_z in terms of the transverse spherical components E_ε, E_α. Let $\hat{r} = \hat{k} = \hat{x}\, k_x + \hat{y}\, k_y + \hat{z}\, k_z$ represent the direction from whence the plane wave emanates, where k_x, k_y, k_z are the direction cosines of $\hat{r} = \hat{k}$. Then, with reference to Figure 7-1, there results

$$\hat{\varepsilon} = \hat{x}\,\frac{-k_x k_y}{\sqrt{1 - k_y^2}} + \hat{y}\sqrt{1 - k_y^2} + \hat{z}\,\frac{-k_y k_z}{\sqrt{1 - k_y^2}} \tag{1}$$

$$\hat{\alpha} = \hat{x}\,\frac{k_z}{\sqrt{1 - k_y^2}} + \hat{y}(0) + \hat{z}\,\frac{-k_x}{\sqrt{1 - k_y^2}} \tag{2}$$

except at $k_y = \pm 1$, where these equations reduce to

$$\hat{\varepsilon} = -\hat{z} \tag{3}$$

$$\hat{\alpha} = \hat{x} \tag{4}$$

The expressions for the field components for the four cases of interest are summarized in Table 7-1. The corresponding magnetic field can be obtained from

$$\underline{H} = (\underline{E} \times \hat{k})/\eta \qquad (\;)$$

[illegible]. Program Flow: Compare expressions in Table 7-1 directly to the program listing below.

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PARAMETRIC INVESTIGATION OF RADOME ANALYSIS METHODS. VOLUME II.--ETC(U)
FEB 81 G K HUDDLESTON, H L BASSETT AFOSR-77-3469
UNCLASSIFIED AFOSR-TR-81-0460 NL

Table 7-1. Rectangular Components for Four Cases of Plane Waves

IOPT	E_x	E_y	E_z
1	$-k_x k_y/\sqrt{1-k_y^2}$	$\sqrt{1-k_y^2}$	$-k_y k_z/\sqrt{1-k_y^2}$
2	$k_z/\sqrt{1-k_y^2}$	0	$-k_x/\sqrt{1-k_y^2}$
3	$(k_z-k_x k_y e^{j\frac{\pi}{2}})/\sqrt{2(1-k_y^2)}$	$e^{j\frac{\pi}{2}}\sqrt{1-k_y^2}/\sqrt{2}$	$(-k_x-k_y k_z e^{j\frac{\pi}{2}})/\sqrt{2(1-k_y^2)}$
4	$(k_z-k_x k_y e^{-j\frac{\pi}{2}})/\sqrt{2(1-k_y^2)}$	$e^{-j\frac{\pi}{2}}\sqrt{1-k_y^2}/\sqrt{2}$	$(-k_x-k_y k_z e^{-j\frac{\pi}{2}})/\sqrt{2(1-k_y^2)}$

7-6. Test Case: See Chapter 2.

7-7. References

1. D. T. Paris and F. K. Hurd, Basic Electromagnetic Theory, McGraw-Hill, New York, 1969, pp. 8-9.

7-8. Program Listing: See following pages.

```
      SUBROUTINE INCPW(KA,EI,IOPT)
C KA=NEGATIVE OF DIR OF PROP'N OF INCIDENT PLANE WAVE (ANT COORD)
C EI= ELECTRIC FIELD VECTOR OF INCIDENT PLANE WAVE (OUTPUT)
C IOPT=1 MAKES EI ELEVATION COMPONENT ONLY
C      =2 MAKES EI AZIMUTH COMPONENT ONLY
C      =3 FOR RHC POLARIZATION (DEFINED WRT DIR OF PROP OF INC WAVE)
C      =4 FOR LHC
C POWER OF INCIDENT WAVE IS UNITY.
      COMPLEX EI(3),CIA
      REAL KA(3),IE,IA
    1 FORMAT(/" ERROR IN SUBR INCPW   IOPT= ",I3//)
C COMPUTE ELEVATION COMPONENT ONLY:
      R=1.-KA(3)**2
      IF (R.LT.0.) R=0.
      R=SQRT(R)
      RY=1.-KA(2)**2
      IF (RY.GT.0.) GO TO 5
      GO TO 40
    5 RY=SQRT(RY)
      GO TO (10,20,30,30),IOPT
C CORRECTIONS TO LOOPS 10,20,30,40 MADE JAN 78 BY GKH.
   10 IE=1.
      EI(1)=-KA(2)*KA(1)*IE/RY
      EI(2)=RY*IE
      EI(3)=-KA(2)*KA(3)*IE/RY
      RETURN
C COMPUTE AZIMUTH COMPONENT ONLY:
   20 IA=1.
      EI(1)=+KA(3)*IA/RY
      EI(2)=CMPLX(0.,0.)
      EI(3)=-IA*KA(1)/RY
      RETURN
C COMPUTE RHC:
   30 IE=.707
      CIA=CMPLX(0.,1.)*IE
      IF (IOPT.EQ.4) CIA=CMPLX(0.,-1.)*IE
      EI(1)=(-KA(2)*KA(1)*CIA-KA(3)*IE)/RY
      EI(2)=CIA*RY
```

```
      EI(3)=(-KA(2)*KA(3)*CIA-IE*KA(1))/RY
      RETURN
   40 GO TO (50,60,70,70),IOPT
   50 EI(1)=(0.,0.)
      EI(2)=(0.,0.)
      EI(3)=-KA(2)
      RETURN
   60 EI(1)=(1.,0.)
      EI(2)=(0.,0.)
      EI(3)=(0.,0.)
   70 IE=.707
      CIA=CMPLX(0.,1.)*IE
      IF (IOPT.EQ.4) CIA=CMPLX(0.,-1.)*IE
      EI(1)=IE
      EI(2)=(0.,0.)
      EI(3)=-KA(2)*CIA
      RETURN
      END
```

Chapter 8

SUBROUTINE RECM

8-1. Purpose: To compute the complex voltages produced at the terminals of the three channels of a radome enclosed monopulse antenna by a plane wave of specified polarization and direction of arrival.

8-2. Usage: CALL RECM (PWI, KA, NX, NY, KXMAX, KYMAX, FGHZ, ROTATE, TRANSL, SUMX, SUMY, DELX, DELY, VR, TABLE, SUPPRS, RSQMAX)

8-3. Arguments

PWI - A complex array of three elements containing E_x, E_y, E_z of the incident plane wave. See Subroutine INCPW.

KA - A real array of three elements containing the direction cosines k_{xA}, k_{yA}, k_{zA} of the unit vector $\hat{k}_A$ which points from the antenna origin in the direction from whence the plane wave emanates.

NX,NY - The even integer number of sample points in x_A and y_A directions used to represent the antenna aperture fields.

KXMAX,KYMAX- Real variables which represent the normalized folding wavenumbers corresponding to the sample distances Δx_A, Δy_A according to $\Delta x_A=\lambda/(2*KXMAX)$, $\Delta y_A=\lambda/(2*KYMAX)$, where λ is the free space wavelength.

FGHZ - Frequency in gigahertz of the monochromatic plane wave.

ROTATE,TRANSL- Real matrices of direction cosines and translation distances used to carry out coordinate transformations of points and vectors from antenna to radome coordinate systems, and vice versa. See Subroutine ORIENT.

SUMX,SUMY - Two dimensional (NX X NY) complex arrays of the x and y vector components of the antenna aperture fields for the sum channel of a three-channel monopulse antenna. The element at I=NX/2+1, J=NY/2+1, corresponds to that at $x_A=0$, $y_A=0$ in the aperture. The general correspondence is given by

$$x_A = - x_{max} + (I-1)*\Delta x_A = (I-MIDNX)*\Delta x_A$$

$$y_A = - y_{max} + (J-1)*\Delta y_A = (J-MIDNY)*\Delta y_A$$

where $x_{max} = \Delta x_A*NX/2$ and $y_{max} = \Delta y_A *NY/2$.

Also see Subroutine HACNF.

DELX,DELY - Antenna aperture fields for the difference elevation channel.

DAZX,DAZY - Antenna aperture fields for the difference azimuth channel.

VR - Complex array of three elements which on output contains the complex terminal voltage of the antenna for the sum, elevation difference, and azimuth difference channels, respectively.

TABLE - Logical variable required by Subroutine RXMIT: if TRUE, a look-up table is used to calculate the transmission coefficients of the radome wall; if FALSE, these coefficients are calculated exactly for each angle of incidence specified.

SUPPRS - Logical variable used to control the printing of results from Subroutine RXMIT: if FALSE, a table of power transmission and reflection coefficients for equal increments in the sine of the incidence angle is printed. The phases of the complex voltage transmission and reflection coefficients of the radome wall are also printed.

RSQMAX - Real variable denoting the maximum radius of the antenna aperture such that any point $(x_A^2+y_A^2)>$RSQMAX is omitted from the ray tracing and summation procedure used to compute the received voltages VR.

8-4. Comments and Method

a. Subroutines Required: TRACE, VECTOR, POINT, RXMIT, CAXB.

b. Method: The voltage V_R induced at the terminals of a linear antenna by a "received" electromagnetic plane wave $\underline{E}_R$, $\underline{H}_R$ is given by the Lorentz reciprocity theorem as [1]

$$V_R(\hat{k}) = C \oint_S (\underline{E}_T \times \underline{H}_R - \underline{E}_R \times \underline{H}_T) \cdot \hat{n} da \qquad (1)$$

where $\hat{k}$ is the unit vector which points in the direction from whence the plane wave emanates and where $\underline{E}_T$, $\underline{H}_T$ are the electromagnetic fields of

the antenna as produced on the closed surface S which surrounds the antenna when it is transmitting. The unit vector $\hat{n}$ is the normal to S pointing into the region not containing the antenna, and C is a complex constant.

When the closed surface S is the z_A=0 plane, $\hat{n}=\hat{z}_A$, $da=dx_A dy_A \simeq \Delta x_A \Delta y_A$ and the integral in (1) can be approximated by

$$V_R(\hat{k}) = C\Delta x_A \Delta y_A \sum_l \sum_m (E_{Tx} H_{Ry} - E_{Ty} H_{Rx} - E_{Rx} H_{Ty} + E_{Ry} H_{Tx}) \quad (2)$$

where Δx_A, Δy_A are equal sample distances in x_A and y_A and where the rectangular vector components of the fields on the z_A=0 plane are given generically by

$$\underline{E}_T = \hat{x}_A E_{Tx} + \hat{y}_A E_{Ty} + \hat{z}_A E_{Tz} \quad (3)$$

It is assumed that the fields $\underline{E}_T$, $\underline{H}_T$ on S with the radome in place are unperturbed by the radome. Also, $\underline{E}_T$ is specified according to the aperture distribution and polarization desired as is usually done in antenna analysis. The corresponding magnetic field $\underline{H}_T$, however, presents something of a vexation in that a non-Maxwellian aperture field can result if $\underline{H}_T$ is specified independently of $\underline{E}_T$ and Maxwell's equation $\underline{H}_T=\nabla x\underline{E}_T/-j\omega\mu$. On the other hand, specification of $\underline{H}_T$ independently of $\underline{E}_T$ is tantamount to specifying magnetic and electric current sheets in the antenna aperture which produce two independent solutions to Maxwell's equations whose sum yields the total fields. This latter approach is

taken here when the geometrical optics approximation

$$\underline{H}_T = \frac{\hat{z}_A \times \underline{E}_T}{\eta} \qquad (4)$$

is utilized, where $\eta=\sqrt{\mu/\varepsilon} \approx 377$ ohms. Also, the magnetic field $\underline{H}_R$ is given by a similar formula

$$\underline{H}_R = \frac{-\hat{k} \times \underline{E}_R}{\eta} \qquad (5)$$

where $-\hat{k}$ is the direction of propagation of the incident plane wave.

Combining the results of Equations (4) and (5) into (2), and designating the origin $x_A=y_A=z_A=0$ as the phase reference for the complex fields, there results

$$V_R(\hat{k}) = C' \sum_l \sum_m \{[(E_{Tx} E_{Rx} + E_{Ty} E_{Ry})(1 + k_{zA}) - (k_{xA} E_{Tx} + k_{yA} E_{Ty})E_{Rz}] \cdot e^{j \frac{2\pi}{\lambda} (k_{xA} x_A + k_{yA} y_A)}\} \qquad (6)$$

where

$$\hat{k} = \hat{x}_A k_{xA} + \hat{y}_A k_{yA} + \hat{z}_A k_{zA} \qquad (7)$$

$$k_{xA}^2 + k_{yA}^2 + k_{zA}^2 = 1 \qquad (8)$$

and where the exponential factor accounts for the phase of the incident wave. It is noted that k_{xA}, k_{yA}, k_{zA} are direction cosines of $\hat{k}$; hence, $k_{zA}=\cos\theta$, where θ is the polar angle measured from the z_A-axis in the usual spherical coordinate system. The $(1+\cos\theta)$ term in Equation (6) is characteristic of the geometrical optics approximation of Equation (4) [2]. The other factors have been absorbed into complex constant C'.

The effects of the radome on the received voltage given by Equation (6) are accounted for by tracing a ray from each aperture element $\Delta x_A \Delta y_A$ in the direction $\hat{k}$ and weighting the field $\underline{E}_R$ associated with the ray by the complex insertion transmission coefficients $T_\perp$, $T_{||}$ of the radome wall as shown in Figure 8-1. These coefficients depend on the incidence angle θ_i and the plane of incidence defined by $\hat{k}$ and the unit inward normal $\hat{n}_R$ to the radome wall at each point of incidence for each ray as illustrated in Figure 8-2. The ray tracing is carried out in the direction $\hat{k}$, and the direction of propagation of each ray is assumed to be the same on both sides of the wall, an assumption that mandates use of the insertion transmission coefficients defined for an infinite sheet by

$$T_\perp = \frac{E_\perp(P)}{E_{\perp i}(P)} \quad (9)$$

$$T_{||} = \frac{E_{||}(P)}{E_{|| i}(P)} \quad (10)$$

where the numerator in each case is the field at point P with the sheet in place and the denominator is the field at the same point with the sheet removed.

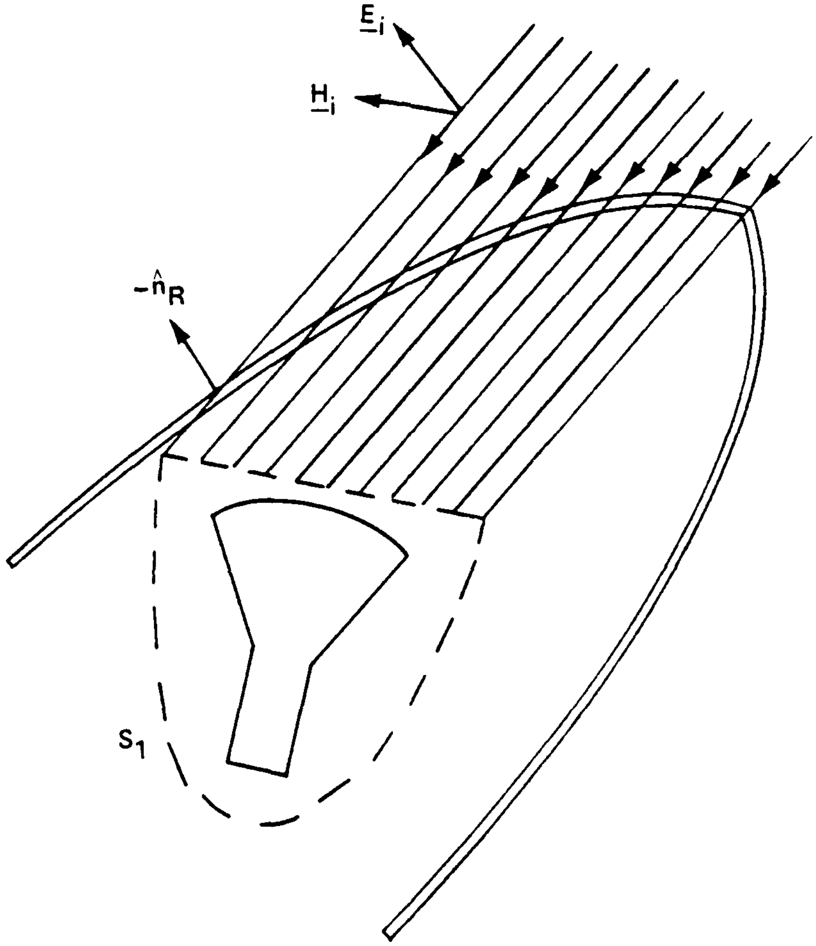

Figure 8-1. Illustration of the Fast Receiving
Method of Radome Analysis

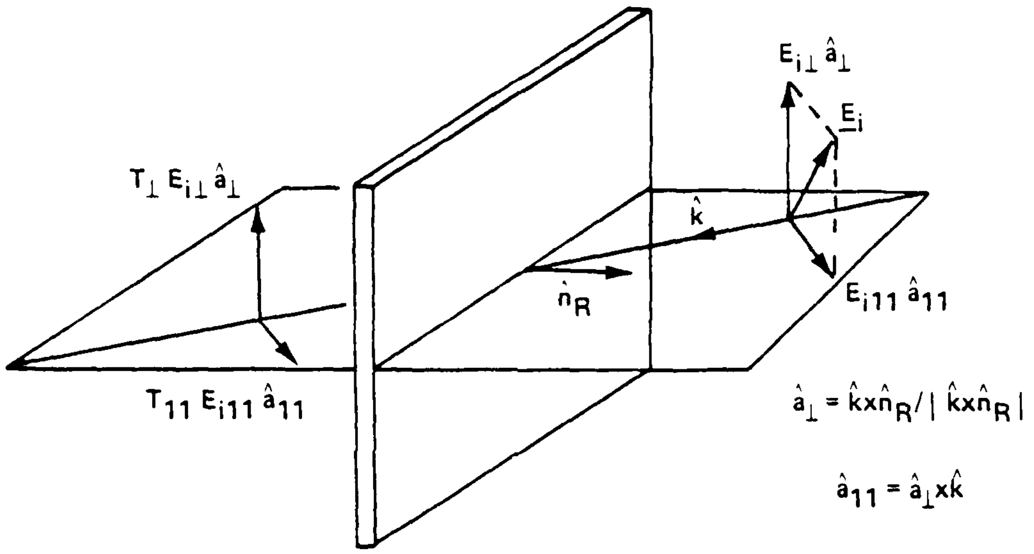

Figure 8-2. Plane Wave Propagation Through an Infinite Plane Sheet

The ray tracing is carried out in radome coordinates (x_R, y_R, z_R), and transformations of points and vectors from antenna coordinates (x_A, y_A, z_A) to radome coordinates, and vice versa, are required. (These transformations are described in detail in Subroutines ORIENT, POINT, and VECTOR.) Let (x_A, y_A, 0) be the point in the aperture from which the ray (line) emanates in the direction $\hat{k}$. Convert this point and unit vector to the radome coordinate system. Find the intersection (x_{RI}, y_{RI}, z_{RI}) of this ray with the inner radome surface as described by $f(\sqrt{x_R^2 + y_R^2}\, z)$ =constant since it is a surface of revolution (Subroutines TRACE and OGIVE). Compute the unit inward normal $\hat{n}_R$ to the surface

$$\hat{n}_R = -\frac{\nabla f}{|\nabla f|} = \hat{x}_R\, n_{xR} + \hat{y}_R\, n_{yR} + \hat{z}_R\, n_{zR} \tag{11}$$

and convert it to antenna coordinates

$$\hat{n}_R = \hat{n}_A = \hat{x}_A\, n_{xA} + \hat{y}_A\, n_{yA} + \hat{z}_A\, n_{zA} \tag{12}$$

Use $\hat{n}_A$ and $\hat{k}$ to determine the plane of incidence, angle of incidence, and the transmitted plane wave $\underline{E}'_R$, $\underline{H}'_R$ (see Subroutine RXMIT) for this ray. Substitute into Equation (6) and sum the results to obtain the following expression for the received voltage

$$V_R(\hat{k}) = C'' \sum_l \sum_m \left[-(1 + k_{zA})(E'_{Rx}\, E_{Tx} + E'_{Ry}\, E_{Ty}) + (k_{xA}\, E_{Tx} + k_{yA}\, E_{Ty}) E'_{Rz} \right]$$

$$\cdot\, e^{j\frac{2\pi}{\lambda}(k_{xA}\, x_A + k_{yA}\, y_A)} \tag{13}$$

where a sign change and η^{-1} have been absorbed into C".

Equation (13) with C"=1 is used in Subroutine RECM to compute the received voltage on each of the three monopulse channels. Note that the received field $\underline{E}_R'$, $\underline{H}_R'$ at each point $(x_A, y_A, 0)$ is the same for all three channels so that three summations can be carried simultaneously to maximize computational speed. In each summation, only the data corresponding to E_{Tx}, E_{Ty} for the sum, elevation difference, and azimuth difference channels need to be changed. Note also that Equation (13) can be rewritten as

$$V_R(\hat{k}) = \sum_l \sum_m [E_{Tx}(\eta H_{Ry}' - E_{Rx}') - E_{Ty} (\eta H_{Rx}' + E_{Ry}')] e^{j \frac{2\pi}{\lambda} (k_{xA} x_A + k_{yA} y_A)} \tag{14}$$

where $\eta H_{Ry}'$, $\eta H_{Rx}'$ are given by Equation (5).

8-5. Program Flow

Line 12: Initialize the ray counter NRAY.

Lines 13-18: Compute λ (cm), $k_o=2\pi/\lambda$, Δx_A, Δy_A, and the midpoint of the NX X NY data arrays corresponding to $x_A=0$, $y_A=0$.

Lines 21-24: Set z_A=0=PA(3) and precalculate $k_o k_{xA}$ and $k_o k_{yA}$. Transform $\hat{k}_A$ to radome coordinates $\hat{k}_R=\hat{x}_R k_{xR} + \hat{y}_R k_{yR} + \hat{z}_R k_{zR}$ in preparation for the ray tracing.

Lines 26-28: Initialize the summations VR(1), VR(2), VR(3) for the received voltages on the sum, difference elevation, and difference azimuth channels, respectively.

Lines 30-33: Iterate for each aperture point x_A=PA(1); precalculate x_A^2 and $k_o k_{xA} x_A$ outside the subsequent loop for y_A.

Lines 34-40: Iterate for each aperture point y_A=PA(2). Compute $x_A^2+y_A^2$=RSQ: if point is outside RSQMAX, omit from computation.

Lines 41-47: Transform $(x_A, y_A, 0)$ to radome coordinates and trace ray to radome inner surface to find unit inward normal $\hat{n}_R$. If metal tip or bulkhead is encountered by ray, omit this ray from computation of received voltages.

Lines 48-52: Transform $\hat{n}_R$ to antenna coordinates and compute the transmitted plane wave PWT=$(E'_{Rx}, E'_{Ry}, E'_{Rz})$.

Lines 53-57: Compute phase PC=$e^{j \frac{2\pi}{\lambda} (k_{xA} x_A + k_{yA} y_A)}$ and apply to $E'_{Rx}, E'_{yR}, E'_{Rz}$.

Lines 58-71: Form $\eta \underline{H}'_R = \underline{E}'_R \times \hat{k}$ and use Equation (14) to add the contribution of this ray to the received voltage on each of the three channels.

Lines 72-73: Increment ray counter and continue the iteration until all aperture points have been used. Upon completion, NRAY equals the number of rays used in the summation for each received voltage.

Lines 74-75: If SUPPRS is false, write NRAY.

RETURN

END

8-6. Test Case: See Chapter 2.

8-7. References

1. G. K. Huddleston, H. L. Bassett, and J. M. Newton, "Parametric Investigation of Radome Analysis Methods," 1978 IEEE APS Symposium Digest, pp. 199-201, May 1978.

2. Microwave Antenna Theory and Design, ed. by S. Silver, McGraw Hill, New York, pp. 161-162, 1949.

8-8. Program Listing: See following pages.

```
      SUBROUTINE RECM(PWI,KA,NX,NY,KXMAX,KYMAX,FGHZ,ROTATE,TRANSL,
     * SUMX,SUMY,DELX,DELY,DAZX,DAZY,VP,TABLE,SUPPRS,RSQMAX)
C  NOTE: HXI,HYI MAGNETIC FIELDS HAVE NOT BEEN DIVIDED BY ETAO.
      REAL KXMAX,KYMAX,LAMBDA,KO,ROTATE(3,3),TRANSL(3)
      REAL KR(3),KA(3),NIR(3),NIA(3),PR(3),PA(3),PIR(3)
      LOGICAL ATOR,RTOA,METAL,TABLE,SUPPRS
      COMPLEX SUMX(NX,NY),SUMY(NX,NY),DELX(NX,NY),DELY(NX,NY)
      COMPLEX DAZX(NX,NY),DAZY(NX,NY),VR(3)
      COMPLEX PWI(3),PWT(3),HPWT(3),PC
      DATA ATOR,RTOA/.TRUE.,.FALSE./
      DATA TUPI/6.2831853071796/,NDC/0/
    1 NRAY=0
      LAMBDA=29.97925/FGHZ
      KO=TUPI/LAMBDA
      DX=LAMBDA/(2*KXMAX)
      DY=LAMBDA/(2*KYMAX)
      MIDNX=NX/2+1
      MIDNY=NY/2+1
C
C  RSQMAX IS THE SQUARE OF THE MAXIMUM RADIUS OF THE APERTURE
      PA(3)=0.
      PHKA1=KO*KA(1)
      PHKA2=KO*KA(2)
      CALL VECTOR(KA,KR,ATOR,ROTATE)
C
      VR(1)=(0.,0.)
      VP(2)=(0.,0.)
      VP(3)=(0.,0.)
C  ITERATE FOR EACH APERTURE POINT
      DO 10 L=1,NX,1
      PA(1)=(L-MIDNX)*DX
    3 APA=PA(1)*PA(1)
      PAKA=PA(1)*PHKA1
      DO 10 M=1,NY,1
      PA(2)=(M-MIDNY)*DY
C
C   IF APERTURE POINT IS OUTSIDE CIRCULAR APERTURE, OMIT FROM CALCULATIO
C
```

```
      RSQ=APA+PA(2)*PA(2)
      IF(RSQ.GT.RSQMAX) GO TO 10
     4 CALL POINT(PA,PR,ATOR,ROTATE,TRANSL)
C
C    TRACE RAY TO FIRST INTERSECTION POINT
C    NOTE: ALL APERTURE POINTS MUST BE CONTAINED WITHIN RADOME.
C
      CALL TRACE(PR,KR,PIR,NIR,METAL)
      IF(METAL) GO TO 10
      CALL VECTOR(NIR,NIA,RTOA,ROTATE)
C
C   TABLE OF XMN COEFS IS FORMED ON FIRST CALL TO XMIT
C
    5 CALL FXMIT(PWI,PWT,KA,NIA,PIR,TABLE,SUPPRS,KO)
      PHASE=PAKA+PA(2)*PHKA2
      PC=CEXP(CMPLX(0.0,+AMOD(PHASE,TUPI)))
      PWT(1)=PWT(1)*PC
      PWT(2)=PWT(2)*PC
      PWT(3)=PWT(3)*PC
C   FORM MAGNETIC FIELD
      CALL CAXB(PWT,KA,HPWT)
C   COMPUTE CONTRIBUTION TO RECEIVED VOLTAGE ON EACH CHANNEL:
C     VR(I)=EYT*HPWT(1)-EXT*HPWT(2)+PWT(1)*HYT
C    $ -PWT(2)*HXT+VR(I)
      VR(1)=VR(1)+SUMX(L,M)*(HPWT(2)-PWT(1))-SUMY(L,M)*
     $ (HPWT(1)+PWT(2))
      VR(2)=VR(2)+DELX(L,M)*(HPWT(2)-PWT(1))-DELY(L,M)*
     $ (HPWT(1)+PWT(2))
      VR(3)=VR(3)+DAZX(L,M)*(HPWT(2)-PWT(1))-DAZY(L,M)*
     $ (HPWT(1)+PWT(2))
C   GEOMETRIC OPTICS APPROXIMATION IS USED ABOVE IN EXPRESSIONS
C   FOR REC"D VOLTAGES  I.E.,HT=ZHAT X ET IN APERTURE.
C   DIVISION BY ETA0 IS NOT DONE TO SAVE COMPUTATION TIME.
      NRAY=NRAY+1
   10 CONTINUE
      NCO=1
      IF (.NOT.SUPPRS) WRITE(6,16) NRAY
   16 FORMAT(" NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =",I10)
```

```
25 RETURN
   END
```

Chapter 9

SUBROUTINE TRACE

9-1. Purpose: To direct the ray tracing to find the intersection of a ray emanating from a point inside the radome and the inner radome surface. All dimensions are in centimeters. Radome coordinates are implied.

9-2. Usage: CALL TRACE (PO, K, P, N, METAL)

COMMON/TRACC/Z2, Z1

9-3. Arguments

PO - Real input array of three elements containing the point PO(x_o, y_o, z_o) from which the ray emanates.

K - Real input array of three elements containing the direction cosines of the ray; i.e., K(1) = k_x, K(2) = k_y, K(3) = k_z.

P - Real output array of three elements containing the point of intersection P(x, y, z) of the ray and the inner radome surface.

N - Real output array containing the direction cosines of the unit inward normal vector to the radome inner surface at P(x, y, z); i.e., N(1) = n_x, N(2) = n_y, N(3) = n_z where $\hat{n} = \hat{x}n_x + \hat{y}n_y + \hat{z}n_z$.

METAL - Logical output variable which indicates any opaque surfaces encountered, such as a metal tip or bulkhead: METAL = .TRUE. indicates that P(x, y, z) lies on such an opaque surface.

Z2 - Real input variable which designates the z_R coordinate (cm) of the intersection of the ogive section of the radome, and the metal tip (if any); must be set in main program prior to the first call to TRACE.

Z1 - Real input variable which designates the z_R coordinate (cm) of the intersection of the ogive section and the bulkhead of the air frame; must be set in main program prior to the first call to TRACE.

9-4. Comments and Method

a. Subroutines required: OGIVE, TDISK, BDISK, OGIVEN, TDISKN, BDISKN

b. The inner surface of the radome is represented by three distinct surfaces as indicated in Figure 9-1: a planar bottom disk (bulkhead), a tangent ogive, and a planar top disk (base of a metal tip). The ray is traced to the ogive surface first to find a point of intersection P(x, y, z):

(1) If $z_1<z<z_2$, then the ogive section of the radome was struck, the unit normal is computed (OGIVEN), METAL is set to .FALSE. and the program returns.

(2) If $z>z_2$, it is assumed that the ray encountered the top disk before impinging on the ogive surface (which actually extends beyond the z_2 coordinate). The ray is then traced to find its intersection with the plane $z = z_2$. If the top disk is indeed struck, then METAL is set .TRUE. and $\hat{n} = -\hat{z}$ is returned.

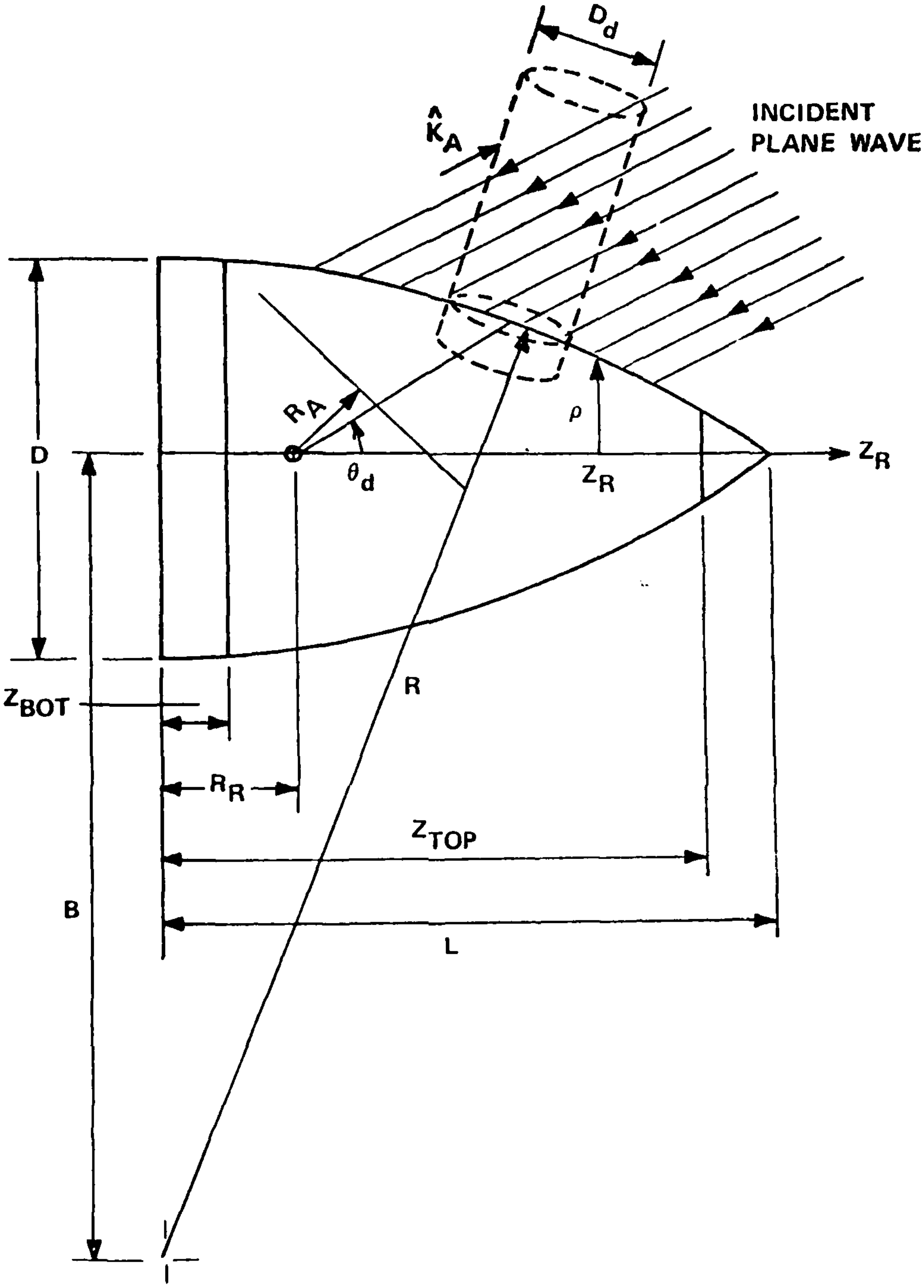

Figure 9-1. Tangent Ogive Radome Geometry.

(3) If $z<z_1$ from (1) above, it is assumed that the bottom disk was struck by the ray before it encountered the ogive surface. The ray is traced again to find its intersection with the plane $z = z_1$. If this bottom disk is indeed struck, then METAL = .TRUE. and $\hat{n} = \hat{z}$ is returned.

The steps in (2) and (3) above appear to be unnecessary; however, they are included to ensure that the ray tracing procedure works correctly and to alert the user if it does not. For example, if incorrect variable values are passed to the supporting subroutines, there is a good chance that no intersection will be found with any one of the three surfaces, in which case the following error message is outputted:

"THERE IS A HOLE IN THIS RADOME"

The message is continued with the values of (x_o, y_o, z_o) and (k_x, k_y, k_z). Incorrect values of geometry variables passed to the supporting subroutines, and attempts to trace a ray from a point exterior to the inner radome surface, will prompt the error message and alert the user of his mistake.

9-5. Program Flow: See listing below.

9-6. See Chapter 2.

9-7. References: None

9-8. Program Listing: See following pages.

```
      SUBROUTINE TRACE(PO,K,P,N,METAL)
C
C     SUBROUTINE TRACE TRACES A RAY FROM ITS POINT OF ORIGIN PO(3)
C     TO ITS POINT OF INTERSECTION WITH THE RADOME WALL, P(3)
C     THE RAY IS TRAVELING IN THE DIRECTION K(3)
C
      REAL PO(3),K(3),P(3),N(3)
      LOGICAL METAL,HIT
C
C   SET SURFACE INTERSECTION Z VALUES
C   Z2 IS THE INTERSECTION OF THE TOP DISK AND THE OGIVE
C   Z1 IS THE INTERSECTION OF THE OGIVE AND THE BOTTOM DISK
      COMMON/TRACC/Z2,Z1
C
C   DETERMINE IF RAY INTERSECTS WITH OGIVE SECTION
      CALL OGIVE(PO,K,P,HIT)
      IF(.NOT.HIT) GO TO 2
      IF(P(3).LT.Z1) GO TO 2
      IF(P(3).GT.Z2) GO TO 1
      GO TO 10
C
C   DETERMINE IF RAY INTERSECTS WITH TOP DISK
    1 CALL TDISK(PO,K,P,HIT)
      IF(HIT) GO TO 11
      GO TO 13
C
C   DETERMINE IF RAY INTERSECTS WITH BOTTOM DISK
    2 CALL BDISK(PO,K,P,HIT)
      IF(HIT) GO TO 12
   13 WRITE(6,100)
  100 FORMAT(/,2X,"THERE IS A HOLE IN THIS RADOME")
      WRITE(6,101)PO,K
  101 FORMAT(2X,"RAY STARTED HERE",3G10.4,"RAY TRAVELED IN THIS DIRECTIO
     &N",3G10.4//)
      RETURN
   10 CALL OGIVEN(P,N)
      METAL=.FALSE.
      RETURN
```

```
11 CALL TDISKN(N)
   METALE=.TRUE.
   RETURN
12 CALL RDISKN(N)
   METALE=.TRUE.
   RETURN
   END
```

Chapter 10

SUBROUTINE RXMIT

10-1. Purpose: To compute the complex rectangular vector components of the electric field $\underline{E}_T$ transmitted through the radome wall, where it is assumed that the incident field $\underline{E}_i$, $\underline{H}_i$ is locally a plane wave and that the radome wall behaves as an infinite plane dielectric sheet. The direction of propagation of the plane wave $-\hat{k}$ and the unit inward normal $\hat{n}$ at the point $P_1(x, y, z)$ are used to determine the angle of incidence and the plane of incidence of the plane wave. All dimensions are in centimeters. Radome coordinates are implied.

10-2. Usage: CALL RXMIT (PWI, PWT, K, NORM, P1, TABLE, SUPPRS, BETA)

COMMON/TRANSC/DIN(6), ER(6), TD(6), TZ, WALTOL, N, NN, D(6), ZB, TK

10-3. Arguments

PWI - Complex input array containing the vector components of the incident electric field; i.e., PWI (E_{xi}, E_{yi}, E_{zi}).

PWT - Complex output array containing the vector components of the transmitted electric field; i.e., PWT (E_{xt}, E_{yt}, E_{zt}).

K - Real input array containing the direction cosines of the direction from whence the plane wave emanates; i.e., $K(k_x, k_y, k_z) = \hat{x}\, k_x + \hat{y}\, k_y + \hat{z}\, k_z$.

NORM - Real input array containing the rectangular components of the unit inward normal $\hat{n} = \hat{x}\, n_x + \hat{y}\, n_y + \hat{z}\, n_z$; i.e., NORM (n_x, n_y, n_z).

PI - Real input array containing the coordinates (x, y, z) of the point on the radome inner surface where the transmitted plane wave is assumed to emerge; i.e., PI(x, y, z).

TABLE - Logical input variable: if TRUE, a look-up table is used to compute the insertion voltage transmission coefficients $T_\perp$, $T_{||}$ corresponding to the angle of incidence θ_i; if FALSE, $T_\perp$, $T_{||}$ are each set to unity to simulate the absence of the radome. Originally, if TABLE = .FALSE., the coefficients $T_\perp$, $T_{||}$ were computed at each point PI(x, y, z) by a call to Subroutine WALL as in the case of the wall configuration being dependent on position (temperature variables, prescription tapers, etc.)

SUPPRS - Logical input variable: if FALSE, a table of transmission coefficients versus $\sin\theta_i$ is printed. Actually, $|T_\perp|^2$, $|T_{||}|^2$, $|R_\perp|^2$, $|R_{||}|^2$ and the phases of $T_\perp$, $T_{||}$, $R_\perp$, $R_{||}$ are printed.

BETA - Real input variable $\beta = 2\pi/\lambda$, where λ is the free space wavelength (cm).

DIN, ER, TD, N, - Real input arrays which specify the thickness in inches, relative dielectric constant ϵ_r, and loss tangent $\tan\delta$ of the N layers comprising the multi-layer radome wall. Layer 1 is the first layer on

N,		the exit side of the wall; layer N is the first
NN		layer on the incident side. ER(NN), TD(NN) specify ε_r, tanδ of the medium in which the N-layer structure is immersed (normally, free space so that ER(NN) = 1.0, TD(NN) = 0.0). The real array D contains, after the first call to RXMIT, the thickness in centimeters of each layer.
TZ, WALTOL, ZB, TK	-	Real variables used previously to specify longitudinal and circumferential variations in wall thickness and in the tolerance on thickness. These variables are not active in this version of RXMIT.

10-4. Comment and Method

a. Subroutines required: WALL, AMPHS, AXB

b. The transmission of an incident plane wave through a plane dielectric sheet immersed in free space (ε_o = 8.854 X 10^{-12} farads/m, μ_o = 4π X 10^{-7} henries/m) may be described in terms of the insertion voltage transmission coefficients

$$T_{\perp} = \frac{E_{t\perp}(P')}{E_{i\perp}(P')} \qquad (1)$$

$$T_{||} = \frac{E_{t||}(P')}{E_{i||}(P')} \qquad (2)$$

where $E_{t\perp}$, $E_{t||}$ are the transmitted fields at P' with the sheet in place, and $E_{i\perp}$, $E_{i||}$ are the incident fields at the same point in the absence of the sheet. The point P' lies on the colinear extension of the incident ray and is located on the exit side of the sheet.

Since the transmission coefficients $T_\perp$, $T_{||}$ are different, it is necessary to resolve the incident electric field $\underline{E}_i$ into perpendicular and parallel components; i.e., vector components which are perpendicular to and parallel to the plane of incidence (POI) defined by $\hat{k}$ and $\hat{n}_R$ as illustrated in Figure 10-1. The unit vector perpendicular to the POI is given by

$$k \quad \frac{\hat{k} \times \hat{n}_R}{|\hat{k} \times \hat{n}_R|} = \frac{\hat{k} \times \hat{n}_R}{\sin|\underline{\hat{k},\hat{n}_R}} \tag{3}$$

A unit vector parallel to the POI is given by

$$\hat{k}_{||} = \hat{k}_\perp \times \hat{k} \tag{4}$$

Th incident electric field may be written as

$$\underline{E}_i = \hat{x}\, E_{xi} + \hat{y}\, E_{yi} + \hat{z}\, E_{iz} = \hat{k}_\perp\, E_{i\perp} + \hat{k}_{||}\, E_{i||} \tag{5}$$

where

$$E_{i\perp} = \hat{k}_\perp \cdot \underline{E}_i = k_{x\perp}\, E_{xi} + k_{y\perp}\, E_{yi} + k_{z\perp}\, E_{zi} \tag{6}$$

$$E_{i||} = \hat{k}_{||} \cdot \underline{E}_i - k_{x||}\, E_{xi} + k_{y||}\, E_{yi} + k_{z||}\, E_{zi} \tag{7}$$

and where $k_{x\perp}$, $k_{x||}$, etc. are the vector components of $\hat{k}_\perp$, $\hat{k}_{||}$. In terms of the coordinate system (x, y, z),

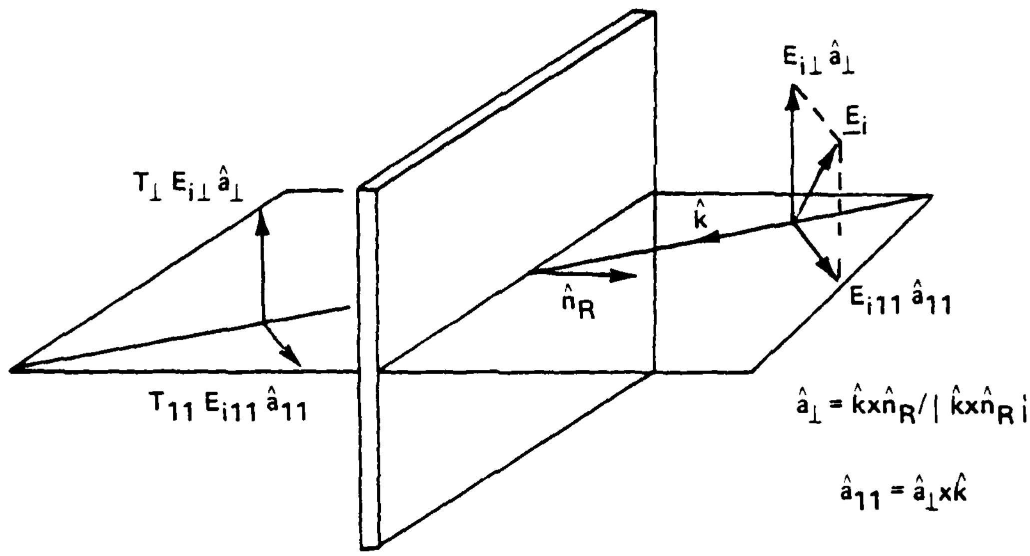

Figure 10-1. Plane Wave Propagation Through an Infinite Plane Sheet

$$\underline{E}_{\perp} = \hat{x}(E_{xi\perp} + E_{xi\parallel}) + \hat{y}(E_{yi\perp} + E_{yi\parallel}) + \hat{z}(E_{zi\perp} + E_{zi\parallel}) \tag{8}$$

where, for example

$$E_{xi\perp} = \hat{x} \cdot \hat{k}_{\perp} E_{i\perp} = k_{x\perp} E_{i\perp} \tag{9}$$

The transmitted plane wave is then given by

$$\underline{E}_T = \hat{k}_{\perp} T_{\perp} E_{i\perp} + \hat{k}_{\parallel} T_{\parallel} E_{i\parallel} \tag{10}$$

$$\underline{E}_T = \hat{x}(T_{\perp} E_{xi\perp} + T_{\parallel} E_{xi\parallel}) + \hat{y}(T_{\perp} E_{yi\perp} + T_{\parallel} E_{yi\parallel}) + \hat{z}(T_{\perp} E_{zi\perp} + T_{\parallel} E_{zi\parallel}) \tag{11}$$

10-5. Program Flow

Lines	Comments
Line 9:	Set NANGLE = number of entries used in the look-up tables for $T_{\perp}$, $T_{\parallel}$.
Lines 10-12:	NDO causes initialization of variables and the computation of the look-up tables on the first call to RXMIT (lines 11-59).
Lines 15-16:	Convert layer thicknesses from inches to centimeters.
Lines 17-59:	Compute look-up tables for $T_{\perp}$, $T_{\parallel}$ at NANGLE points spaced equally in $\sin\theta_i$ over the range (0, 1). If SUPPRS = .FALSE., print a table of transmission coefficients (every fifth point only). If ER(1) < 1.05, set AIR = .TRUE. and compute unity transmission coefficients for the "air" radome (for testing).

Lines 60-78: Compute $\sin\theta_i$.

Lines 79-86: Interpolate in table to compute $T_\perp$, $T_{||}$ at $\sin\theta_i$.

Lines 87-100: Normalize the vector $\hat{k}_\perp$.

Lines 101-112: Compute $E_{xi\perp}$, $E_{yi\perp}$, $E_{zi\perp}$.

Lines 113-124: Compute $E_{xi||}$, $E_{yi||}$, $E_{zi||}$.

Lines 125-129: Compute E_{xt}, E_{yt}, E_{zt} and return.

Lines 130-136: If $\sin\theta_i$ is out of range of the table, write error message, set $T_\perp$, $T_{||}$ to unity, and continue.

10-6. Test Case: See Chapter 2.

10-7. References: None

10-8. Program Listing: See following pages.

```
      SUBROUTINE RXMIT(PWI,PWT,K,NORM,P1,TABLE,SUPPRS,BETA)
      COMPLEX PWI(3),PWT(3),EZPER,EZPAR
      COMPLEX EXPAR,EYPAR,EXPER,EYPER,TPARI,TPERI,DOT,RPERI,RPARI
      REAL K(3),NORM(3),KPER(3),P1(3),AMP(4),PHS(4),KPAR(3)
      LOGICAL TABLE,SUPPRS,AIR
      COMPLEX TPER(250),TPAR(250),RPER,RPAR
      COMMON/TRANSC/DIN(6),ER(6),TD(6),TZ,WALTOL,N,NN,D(6),ZB,TK
      DATA PI/3.14159265/
      DATA NANGLE/250/
      DATA NDO/0/
      IF (NDO.EQ.1) GO TO 5
      NDO=1
      AIR=.FALSE.
      IF (ER(1).LT.1.05) AIR=.TRUE.
      DO 90 I=1,NN
   90 D(I)=DIN(I)*2.54
      PIO2=PI/2.
C
C     FORM WALL TRANSMISSION TABLES
C
      MANGLE=NANGLE-1
      ANGLE=MANGLE
      RAD=180./PI
      IF (.NOT.SUPPRS) WRITE(6,115)
  115 FORMAT(/" ANGLE",7X,"TPERI**2",8X,"TPARI**2",8X,
     $ "RPERI**2",8X,"RPARI**2"//)
  116 FORMAT(1X,F5.2,4(3X,F5.3,F8.1))
      DO 100 I=1,MANGLE
      SINE=(I-1)/ANGLE
      IF (AIR) GO TO 91
      CALL WALL(BETA,SINE,D,ER,TD,N,NN,TPER(I),TPAR(I),RPER,RPAR)
      GO TO 92
   91 TPER(I)=(1.,0.)
      TPAR(I)=(1.,0.)
      RPER=(0.,0.)
      RPAR=(0.,0.)
   92 IF (MOD(I,5).NE.0) GO TO 100
      ANG=ASIN(SINE)*RAD
```

```
      CALL AMPHS(TPER(I),AMP(1),PHS(1))
      CALL AMPHS(TPAR(I),AMP(2),PHS(2))
      CALL AMPHS(RPER,AMP(3),PHS(3))
      CALL AMPHS(RPAR,AMP(4),PHS(4))
C CONVERT TO POWER XMN COEFFICIENTS
      DO 95 L=1,4
   95 AMP(L)=AMP(L)**2
      IF (.NOT.SUPPRS) WRITE(6,116) ANG,((AMP(J),PHS(J)),J=1,4)
  100 CONTINUE
      XC=0.
      IF (ER(1).LT.1.05) XC=1.0
      TPER(NANGLE)=CMPLX(XC,0.)
      TPAR(NANGLE)=CMPLX(XC,0.)
      ANG=90.
      CALL AMPHS(TPER(NANGLE),AMP(1),PHS(1))
      CALL AMPHS(TPAR(NANGLE),AMP(2),PHS(2))
      CALL AMPHS(RPER,AMP(3),PHS(3))
      CALL AMPHS(RPAR,AMP(4),PHS(4))
      IF (.NOT.SUPPRS) WRITE(6,116) ANG,((AMP(J),PHS(J)),J=1,4)
      IF (.NOT.SUPPRS) WRITE(6,105)
  105 FORMAT(//" TABLE OF XMN COEF. IS FORMED"//)
    5 CONTINUE
C
C    FIND VECTOR NORMAL TO NORM AND K
C
      CALL AXB(K,NORM,KPER)
C
C    FIND MAGNITUE OF KPER (THIS IS ALSO THE SINE OF THE INCLUDED ANGLE)
C
      SINE=SQRT(KPER(1)*KPER(1)+KPER(2)*KPER(2)+KPER(3)*KPER(3))
      IF(SINE.GT.1.0) SINE = 1.0
      IF(TABLE) GO TO 25
      TPERI=(1.,0.)
      TPARI=(1.,0.)
      RPERI=(0.,0.)
      RPARI=(0.,0.)
      GO TO 3
C
```

```
C   USE TABLE OF TRANSMISSION COEFFICIENTS
C
  25 RI=SINE*MANGLE+1.C
     IL=RI
     IF((IL.GE.NANGLE).OR.(IL.LT.1)) GO TO 50
     IH=IL+1
     X=RI-IL
     TPERI=(1.0-X)*TPER(IL)+X*TPER(IH)
     TPARI=(1.C-X)*TPAR(IL)+X*TPAR(IH)
C
C   TEST FOR NORMAL INCIDENCE
C
   3 IF(SINE.LT.1F-10) GO TO 2
C
C   UNITIZE PERPENDICULAR VECTOR
C
     SEC=1/SINE
     KPER(1)=KPER(1)*SEC
     KPER(2)=KPER(2)*SEC
     KPER(3)=KPER(3)*SEC
     GO TO 1
   2 KPER(1)=1.0
     KPER(2)=0.0
     KPER(3)=0.C
   1 CONTINUE
C
C   FIND DOT PRODUCT OF INCIDENT ELECTRIC FIELD WITH KPER
C
     DOT=PWI(1)*KPER(1)+PWI(2)*KPER(2)+PWI(3)*KPER(3)
C
C   FIND PERPENDICULAR COMPONENTS OF ELECTRC FIELD
C
     EXPER=DOT*KPER(1)
     EYPER=DOT*KPER(2)
     EZPER=DOT*KPER(3)
C
C   FIND PARALLEL COMPONENTS OF ELECTRIC FIELD
C
```

```
C     EXPAR=PWI(1)-EXPEP
C     EYPAR=PWI(2)-EYPEP
C     EZPAR=PWI(3)-EZPER
      CALL AXB(KPER,K,KPAR)
C  KPAR IS A UNIT VECTOR AS REQUIRED
      DOT=PWI(1)*KPAR(1)+PWI(2)*KPAR(2)+PWI(3)*KPAR(3)
      EXPAR=DOT*KPAR(1)
      EYPAR=DOT*KPAR(2)
      EZPAR=DOT*KPAR(3)
C
C   FIND X AND Y COMPONENTS OF TRANSMITTED FIELD
C
      PWT(1)=EXPAR*TPARI+EXPER*TPERI
      PWT(2)=EYPAR*TPARI+EYPER*TPERI
      PWT(3)=EZPAR*TPARI+EZPER*TPERI
      RETURN
   50 WRITE(6,55) SINE
   55 FORMAT(/10X,"SINE="F10.7," IS NOT IN THE WALL TABLE "/)
      TPERI=(1.,0.)
      TPARI=(1.,0.)
      GO TO 3
      END
```

Chapter 11

SUPROUTINE WALL

11-1. Purpose: To compute the transmission and reflection coefficients of a N-layer dielectric sheet having thicknesses d_n, dielectric constants ε_{rn}, and loss tangents $\tan\delta_n$ for each layer when a plane wave is incident at angle θ_i.

11-2. Usage: CALL WALL (BETA, SINE, D, ER, TD, N, NN, TPER, TPAR, RPER, RPAR)

11-3. Arguments

BETA	-	Real input variable = $2\pi/\lambda$, where λ is the free space wavelength.
SINE	-	Real input variable = $\sin\theta_i$.
D, ER, TD	-	Real input arrays containing the thickness (cm), dielectric constant ε_r, and loss tangent $\tan\delta$ of each layer.
N	-	Integer input variable equal to the number of layers.
NN	-	Integer input = N+1.
TPER,TPAR	-	Complex output variables equal to the insertion voltage transmission coefficients for the components of the incident electric field perpendicular to and parallel to the plane of incidence, respectively.
RPER,RPAR	-	Complex output variables equal to the reflection coefficients $R_\perp$, $R_\parallel$.

11-4. Comment and Method

a. Layer 1 is the first layer on the exit side of the panel; layer N is the first layer on the incident side. $T_{\perp}$, $T_{\parallel}$ have the same value for either side of the panel being the incident side; however, $R_{\perp}$, $R_{\parallel}$ are different (in phase) for the two cases.

b. The details of the method are presented in Reference 1 and are reproduced in Appendix E.

11-5. Program Flow: See Reference 1.

11-6. Test Case: See Chapter 2.

11-7. References

1. E. B. Joy and G. K. Huddleston, "Radome Effects on the Performance of Ground Mapping Radar," Technical Report, Contract DAAH01-72-C-0598, U. S. Army Missile Command, March 1973.

11-8. Program Listing: See following pages.

```
      SUBROUTINE WALL(BETA,SINE,D,ER,TD,N,NN,TPER,TPAR,RPER,RPAR)
C    SUBROUTINE WALL COMPUTES THE TRANSMISSION AND REFLECTION
C    COEFFICIENTS FOR AN N LAYER, PLANE DIELECTRIC PANEL FOR PLANE
C    WAVE INCIDENT AT SINE(ANGLE) FOR PERPENDICULAR AND
C    PARALLEL POLARIZATIONS.
C    PARAMETERS OF THE WALL: N= THE NUMBER OF LAYERS
C                            NN= N+1 REQUIRED TO DIMENSION ARRAYS
C                            D= THICKNESS OF EACH LAYER IN CENTIMETERS
C                            ER= RELATIVE DIELECTRIC CONSTANT OF EACH LAY
C                            TD= THE LOSS TANGENT FOR EACH LAYER
C TN1,TN2 ARE THE NORMAL VOLTAGE XMN COEFFICIENTS: TPER,TPAR ARE THE
C INSERTION VOLTAGE TRANSMISSION COEFFICIENTS. IT IS IMPORTANT TO
C NOTE THAT THE XMN COEFS ARE THE SAME FOR PLANE WAVE INCIDENT FROM
C EITHER SIDE OF THE STRATIFIED DIELECTRIC PANEL IMMERSED IN FREE SPACE:
C HOWEVER, THE REFLECTION COEFS ARE NOT. THAT IS, FOR COMPUTING RPER,
C RPAR, THE ORDERING OF ER(NN),TD(NN) IS IMPORTANT WITH LAYER 1 BEING
C THE FIRST LAYER ON THE EXIT SIDE, LAYER N BEING THE FIRST LAYER ON THE
C INCIDENT SIDE.  LAYER NN  AND  LAYER 0  ARE JUST FREE SPACE  LAYERS
C OF SEMI-INFINITE DEPTH.
C E,G,R1,R2, ARE ARRAYS USED IN THE SUBROUTINE HAVING NN DIM"L LIMITS
      COMPLEX E(6),G(6),R1(6),R2(6),GG,EE,RR1,RR2,AA1,AA2,X1,X2,
     &X3,X4,Y1,Y2,Y3,Y4,U1,U2,U3,U4,V1,V2,V3,V4,P1,P2,P3,P4,Q1,Q2,Q3,Q4
      COMPLEX TPER,TPAR,RPER,RPAR,U,V,TN1,TN2
      DIMENSION ER(NN),TD(NN),D(N)
      ER(NN)=1.0
      TD(NN)=0.
      DO 50 I=1,NN
   50 E(I)=CMPLX(ER(I),-ER(I)*TD(I))
      AB=BETA*0.70710707707071
C
C    CALCULATE TOTAL THICKNESS OF WALL IN CM
C
      DTOTAL=0.0
      DO 200 I=1,N
  200 DTOTAL=DTOTAL+D(I)
C    S IS THE SINE OF THE ANGLE SQUARED
C    C IS THE COSINE OF THE ANGLE
      S=SINE*SINE
```

```
      D=SQRT(1.0-D)
      AD=ER(1)-S
      ET=ER(1)*TL(1)
      SR=SQRT(AD*AD+ET*ET)
      IF(SR-AD) 76,76,77
   76 A=0.
      GO TO 78
   77 A=AR*SQRT(SR-AD)
   78 B=AR*SQRT(SR+AD)
      G(1)=CMPLX(A,B)
      GG=CMPLX(0.0,BETA*C)
      EE=1.0
      SUM=0.
      SUM=SUM+D(1)/SQRT(AD)
      RR1=(G(1)-GG)/(G(1)+GG)
      RR2=(EE*G(1)-E(1)*GG)/(EE*G(1)+E(1)*GG)
      DO 84 I=1,N
      II=I+1
      AD=ER(II)-S
      ET=ER(II)*TD(II)
      IF (I-N) 176,177,177
  176 SUM=SUM+D(II)/SQRT(AD)
  177 CONTINUE
      SR=SQRT(AD*AD+ET*ET)
      IF(SR-AD) 79,79,80
   79 A=0.
      GO TO 81
   80 A=AR*SQRT(SR-AD)
   81 B=AR*SQRT(SR+AD)
      G(II)=CMPLX(A,B)
      R1(I)=(G(II)-G(I))/(G(II)+G(I))
   84 R2(I)=(E(I)*G(II)-E(II)*G(I))/(E(I)*G(II)+E(II)*G(I))
      SUM=S*SUM
      AA1=1.0-RR1
      AA2=1.0-RR2
      DO 85 I=1,N
      AA1=AA1*(1.0-R1(I))
   85 AA2=AA2*(1.0-R2(I))
```

```
AA1=1.0/ AA1
AA2=1.0/ AA2
U=-G(1)*H(1)
V=-(1)*D(1)
X1=C [illegible](U)
X4=CEXP(V)
X2=-R-1*X4
X3=-R-1*X1
Y1=X1
Y4=X4
Y2=-RR2*X4
Y3=-RR1*Y1
DO 100 I=2,NN
IF(I-N.1) [illegible]
U1=1.
U2=-R1(N)
U3=-[illegible]( )
U4=1.0
V1=1.0
V2=-[illegible]( )
V3=-[illegible](N)
V4=1.0
GO TO 1
II=I-1
U=-G(I)*
V=G(I)*
U1=C [illegible]
U4=[illegible]
U2=-[illegible](II)*[illegible]
U3=-R1(II)*[illegible]
V1=[illegible]
[illegible]
V[illegible]=-[illegible](II)*[illegible]
V2=-[illegible](II)*[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
```

```
      Q1=Y1*V1+Y2*V3
      Q2=Y1*V2+Y2*V4
      Q3=Y3*V1+Y4*V3
      Q4=Y3*V2+Y4*V4
      X1=P1
      X2=P2
      X3=P3
      X4=P4
      Y1=Q1
      Y2=Q2
      Y3=Q3
  105 Y4=Q4
      RPER=-X3/X4
C     TN1,TN2 ARE NORMAL VOLTAGE XMN COEFFICIENTS.
      RPAR=-Y3/Y4
      TN1=(X1+X2*RPER)*AA1
      U=CMPLX(0.0,-SUM*BETA)
      U=CEXP(U)
C     TPER,TPAR HERE ARE VOLTAGE XMN COEFFICIENTS AT EXIT POINT OF RAY.
      TPER=TN1*U
      TN2=(Y1+Y2*RPAR)*AA2
      TPAR=TN2*U
C   MODIFY TRANSMISSION COEFFICIENTS FOR INSERTION
      U=CMPLX(0.0,BETA*DTOTAL*C)
      U=CEXP(U)
      TPER=TN1*U
      TPAR=TN2*U
    1 CONTINUE
  300 RETURN
      END
```

Chapter 12

SUBROUTINE AXB

12-1. Purpose: To compute the real vector cross product $\underline{C} = \underline{A} \times \underline{B}$, where $\underline{A}$, $\underline{B}$, $\underline{C}$ are expressed in rectangular components.

12-2. Usage: CALL AXB(A, B, C)

12-3. Arguments

A, B - Real input arrays containing the rectangular components of $\underline{A} = \hat{x} A_x + \hat{y} A_y + \hat{z} A_z$ and $\underline{B}$; i.e., $A(A_x, A_y, A_z)$, $B(B_x, B_y, B_z)$.

C - Real output array containing the rectangular components of the vector $\underline{C} = \underline{A} \times \underline{B}$; i.e., $C(C_x, C_y, C_z)$.

12-4. Comment and Method

a. Both input vectors must be real.

b. The computation of $\underline{C} = \underline{A} \times \underline{B}$ is elementary.

12-5. Program Flow: See listing below.

12-6. Test Case: None

12-7. References: None

12-8. Program Listing: See following page.

```
      SUBROUTINE AXB(A,B,C)
C     COMPUTE VECTOR [illegible] PRODUCT OF A CROSS B, RESULTING IN C
      DIMENSION A(3),B(3),C(3)
      C(1)=A(2)*B(3)-A(3)*B(2)
      C(2)=A(3)*B(1)-A(1)*B(3)
      C(3)=A(1)*B(2)-A(2)*B(1)
      RETURN
      END
```

Chapter 13

SUBROUTINE CAXB

13-1. Purpose: To compute the complex vector cross product $\underline{C} = \underline{A} \times \underline{B}$, where $\underline{A}$ is a complex vector and $\underline{B}$ is a real vector expressed in rectangular coordinates.

13-2. Usage: CALL CAXB (A, B, C)

13-3. Arguments

A - Complex input array containing the rectangular components of the vector $\underline{A} = \hat{x} A_x + \hat{y} A_y + \hat{z} A_z$; i.e., A (A_x, A_y, A_z).

B - Real input array B (B_x, B_y, B_z) representing the vector $\underline{B}$.

C - Complex output array C (C_x, C_y, C_z) representing the vector $\underline{C} = \underline{A} \times \underline{B}$.

13-4. Comment and Method: None

13-5. Program Flow: See listing below.

13-6. Test Case: None

13-7. References: None

13-8. Program Listing: See following page.

```
      SUBROUTINE CAXB(A,B,C)
C CAXB COMPUTES VECTOR CROSS PRODUCT AXB=C, WHERE A AND C
C ARE COMPLEX AND B IS REAL.
      COMPLEX A(3),C(3)
      REAL B(3)
      C(1)=A(2)*B(3)-A(3)*B(2)
      C(2)=A(3)*B(1)-A(1)*B(3)
      C(3)=A(1)*B(2)-A(2)*B(1)
      RETURN
      END
```

Chapter 14

SUBROUTINE RECBS

14-1. Purpose: To compute the angle of arrival $\hat{k}$ of a plane wave on a monopulse antenna which yields an electrical boresight indication which, due to the radome, may be different from the mechanical boresight along the z axis of the antenna. The antenna aperture lies in the xy plane. All dimensions are in centimeters. Antenna coordinates are implied.

14-2. Usage: CALL RECBS (SUMX, SUMY, DELX, DELY, DAZX, DAZY, NX, NY, LMAX, NS, IOPT, VR, DMRAD, ROTATE, TRANSL, FGHZ, KXMAX, KYMAX, TABLE, SINOS, K, AZTM, ELTM, RSQMAX, VMAX, SMAX, SUPPRS)

14-3. Arguments

SUMX, SUMY, DELX, DELY, DAZX, DAZY	-	Complex input arrays of NX by NY elements each containing the aperture distributions of the monopulse antenna. See Subroutine HACNF.
LMAX	-	Integer input variable which controls the maximum number of iterations that will be done to find the electrical boresight within the tolerance specified by DMRAD.
NS	-	Inactive integer variable.
IOPT	-	Integer input variable which selects the polarization of the incident plane wave. See Subroutine INCPW.
VR	-	Complex array of three elements representing the received voltage on the sum, elevation difference,

and azimuth difference channels of the antenna, respectively (output).

DMRAD - Real input variable equal to the tolerance to which the electrical boresight in milliradians is to be computed; i.e., 0.1 milliradian.

ROTATE, TRANSL, FGHZ, KXMAX, KYMAX, TABLE - Variables required by Subroutine RECM. See Chapter 8.

SINOS - Real variable equal to the sine of the angle θ_{os} (measured from the z-axis) in the $\phi = 45°$ plane (ϕ measured from +x toward +y) at which the first target return arrives; e.g., θ_{os} = 3 degrees.

K - Real output array containing the direction of arrival of the final target return; i.e., $K(k_x, k_y, k_z)$.

AZTM, ELTM - Real output variables equal to angles (mrad) in the azimuth and elevation planes of the antenna which specify the direction of arrival of the final target return. If $\hat{k}$ is the unit vector pointing from the origin in the direction of the final return, then the orthographic projection of this vector onto the xz-plane makes an angle AZTM with the z-axis; its projection onto the yz-plane makes an angle ELTM with the z-axis.

RSQMAX - Real variable needed by Subroutine RECM. See Chapter 8.

VMAX - Unused.

SMAX - Real output variable equal to the magnitude of the received sum voltage for the final return; used to compute loss in antenna gain.

SUPPRS - Logical input variable which controls the computation and printing of additional antenna outputs around the boresight direction. If TRUE, the complex voltage outputs of the difference channels will be computed at one milliradian increments over the range ±3.0 mrad, centered on the direction of the final target return.

14-4. Comments and Method

a. Subroutines required: AMPHS, RECM, INCPW.

b. Subroutine RECBS uses a linear tracking model to determine the direction of arrival $\hat{k} = \hat{x}\, k_x + \hat{y}\, k_y + \hat{x}\, k_z$ of a plane wave which will produce null indications in the elevation and azimuth difference channels of the monopulse antenna inside the radome. Subroutine RECM is used to compute the received voltage on each channel for the specified polarization (IOPT) and direction of arrival.

The first target return is made to arrive from the direction

$$\hat{k}_1 = \hat{x}\, \sin\theta_{os} + \hat{y}\, \sin\theta_{os} + \hat{z}\sqrt{1-2\sin^2\theta_{os}} \qquad (1)$$

to produce outputs

$$U_{AZ1} = \mathrm{Im}\left\{\frac{V_{\Delta AZ}}{V_{\Sigma}}\right\} \qquad (2a)$$

$$U_{EL1} = \mathrm{Im}\left\{\frac{V_{\Delta EL}}{V_{\Sigma}}\right\} \qquad (2b)$$

where V_{Σ}, $V_{\Delta EL}$, $V_{\Delta AZ}$ are the complex voltage outputs of the three-channel outputs of the three-channel antenna. The second return is made to arrive from

$$\hat{k}_A = \hat{x}(-\sin\theta_{os}) + \hat{y}(-\sin\theta_{os}) + \hat{z}\sqrt{1-2\sin^2\theta_{os}} \qquad (3)$$

to produce outputs U_{AZ2}, U_{EL2}.

Construct a linear model for each channel independently using the slope/intercept equation for a line; i.e.,

$$U_{AZ} = M_{AZ}\ k_x + b_{AZ} \qquad (4a)$$

$$U_{EL} = M_{EL}\ k_y + b_{EL} \qquad (4b)$$

where

$$M_{AZ} = (U_{AZ1} - U_{AZ2})/\ (k_{x1} - k_{x2}) \qquad (5a)$$

$$M_{EL} = (U_{EL1} - U_{EL2})/(k_{y1} - k_{y2}) \qquad (5b)$$

$$b_{AZ} = U_{AZ1} - M_{AZ}\ k_{x1} \qquad (6a)$$

$$b_{EL} = U_{EL1} - M_{EL}\ k_{y1} \qquad (6b)$$

Use this model to estimate the values of k_x, k_y that will make $U_{AZ} = U_{EL} = 0$; i.e.,

$$k_x = - b_{AZ}/M_{AZ} \quad (7a)$$

$$k_y = - b_{EL}/M_{EL} \quad (7b)$$

where the value of k_z follows from

$$k_x^2 + k_y^2 + k_z^2 = 1 \quad (8)$$

The third target return is made to arrive from this direction and the values of U_{AZ}, U_{EL} are computed via Subroutine RECM. Now, according to the last linear model, a value of U_{AZ} within the range

$$|U_{AZ}| < |M_{AZ} \sin\theta_{tol} + b_{AZ}| \quad (9)$$

would indicate that the null has been found within the tolerance θ_{tol} (=DMRAD) specified. If this tolerance is not satisfied for both channels, then the process is repeated until it is or until LMAX is exceeded. In continuing the iterations, $\hat{k}_2$ becomes $\hat{k}$, and the estimated point becomes $\tilde{k}_2$.

On the last return, the direction of arrival in specified by $\hat{k}$. The angles in the azimuth and elevation planes are given in milliradians by

$$AZTM = \sin^{-1}\left(\frac{k_x}{1-k_y^2}\right) \cdot 1000 \quad (10)$$

$$\mathrm{[illegible]} = \tan^{-1}\left(\frac{k_{[illegible]}}{1-k_x}\right) \cdot [illegible] \qquad ([illegible])$$

[illegible] error slopes, M_{AZ}, M_{EL}, are also [illegible]

[illegible]

$$\mathrm{MESAZ} = M_{AZ} \cdot 57.3 \qquad ([illegible])$$

$$\mathrm{MESEL} = M_{EL} \cdot 57.3 \qquad ([illegible])$$

where the maximum amplitude [illegible]MAX received [illegible] the sum channel [illegible] assumed [illegible] for normalization purposes.

[illegible] additional outputs around the [illegible] are computed. The directions are [illegible] by the angl[illegible]

$$[illegible] = x([illegible] + k_x) + y(\sin[illegible] + k_y) + [illegible]\, k_z \qquad (12)$$

where [illegible] varies over the range [illegible] mrad. At each direction, the monopulse outputs U_{AZ}, U_{EL} are printed as well as the complex monopulse tracking function shown in the brackets of Equations ([illegible]). It is noteworthy that the [illegible] of the tracking function will change from ~-[illegible]° to ~+[illegible]° as the angle [illegible] in Equation (12) goes from negative to positive values. This behavior is a consequence of the phasing chosen for the aperture distributions for the difference channels in Subroutine HACNF.

13-[illegible]. Program Flow

<u>Line Nos.</u> <u>Comments</u>

Lines 11-15: Initialize variables. Convert DMRAD to radians and compute sine.

Lines 16-30: Compute first two target returns to construct linear tracking model.

Lines 31-38: Compute slopes M_{AZ} = SLPAZ, M_{EL} = SLPEL from first two returns.

Line 39: Iterate on linear model up to LMAX times.

Lines 43-44: If the increment in Δk is larger than sin(DMRAD/1000), then use it to compute slopes; if not, use the last computed values of slopes to avoid division by too small a number.

Lines 4[illegible]-46: Compute intercepts b_{AZ}, b_{EL}.

Lines 47-48: Compute accuracy criteria based on current slopes and intercepts.

Lines 4[illegible]-51: Compute direction $\hat{k}$ that the model indicated will produce nulls $U_{AZ} \approx 0$, $U_{EL} \approx 0$ in both planes.

Lines [illegible]-56: Compute U_{EL}, U_{AZ} for this direction $\hat{k}$.

Lines [illegible]7-[illegible]: Update the linear tracking model by storing the last two points in each channel as U(1), U(2); e.g., $U_{AZ}(1)$ $U_{AZ}(2)$ and $U_{AZ}(2)$ = U_{AZ}, $K_1(1) = K_2(1)$ and $K_2(1)$ K(1), etc.

Line [illegible]3: At least three iterations are always used.

Lines [illegible]4-[illegible]5: If U_{AZ}, U_{EL} are within error bounds, exit the loop; if not, continue to iterate.

Lines [illegible]6-[illegible]7: If LMAX exceeded, inform the user.

Line [illegible]: Compute amplitude SMAX on sum channel for final target return.

Lines [illegible]-1[illegible]: Compute slopes for final return.

Lines 1[illegible]1-1[illegible]: Compute boresight error AZTM, ELTM.

Lines 103-104: Compute k_{Σ} for $\hat{k}_1$ and $\hat{k}_2$.

Lines 105-108: Convert slopes to volts/degree.

Lines 109-117: If SUPPRS = .FALSE, print results.

Lines 118-139: Compute and print additional outputs around the boresight direction $\dot{k}$.

Lines 140-144: Compute and print the slopes of a linear tracking model based on the points at +3 mrad and -3 mrad (hence, the division by .006 = 6 mrad).

Line 145: RETURN

Line 146: END

14-6. Test Case: See Chapter 2.

14-7. References: None.

```
      SUBROUTINE PRBMS(SUMX,SUMY,DELX,DELY,DAZX,DAZY,NX,
     &NY,EMAX,NS,IOPT,JK,CMRAD,ROTATE,TRANSL,FGHZ,KXMAX,KYMAX,
     & TABLE,SINCS,K,AZTM,ELTM,RSQMAX,VMAX,SMAX,SUPPRS)
C CMRAD IS THE SPECIFIED ACCURACY OF BORESIGHT DATA (INPUT IN MRAD)
      COMPLEX SUMX(NX,NY),SUMY(NX,NY),DELX(NX,NY),DELY(NX,NY)
      COMPLEX DAZX(NX,NY),DAZY(NX,NY),VP(3)
      COMPLEX EINC(3)
      REAL ROTATE(3,3),TRANSL(3),KXMAX,KYMAX
      REAL K1(3),K2(3),UAZ(2),UEL(2),K(3)
      LOGICAL TABLE,SUPPRS
      CRAD=CMRAD/1000.
      SCRAD=SIN(CRAD)
      SINOFF=0.
      CALFA=0.
      AMUL=1
      DO 70 J=1,2
      IF (J.EQ.2) AMUL=-1.
      K2(1)=SINCS*AMUL
      K2(2)=SINCS*AMUL
C   COMPUTE UAZ,UEL AT K2:
      K2(3)=SQRT(1.-K2(1)**2-K2(2)**2)
      CALL INCPW(K2,EINC,IOPT)
      CALL RECM(EINC,K2,NX,NY,KXMAX,KYMAX,FGHZ,ROTATE,TRANSL,
     & SUMX,SUMY,DELX,DELY,DAZX,DAZY,VP,TABLE,SUPPRS,RSQMAX)
      UAZ(J)=AIMAG(VP(3)/VP(1))
      UEL(J)=AIMAG(VP(2)/VP(1))
   70 CONTINUE
      K1(1)=SINCS
      K1(2)=SINCS
      K1(3)=SQRT(1.-K1(1)**2-K1(2)**2)
      LOOP=0
C   ENSURE THAT INITIAL ESTIMATES MEET PROPER CONDITIONS:
      IF ((UAZ(1).LT.0.).OR.(UAZ(2).GT.0.).OR.(UEL(1).LT.0.).OR.
     &(UEL(2).GT.0.)) GO TO 101
      C=K1(1)-K2(1)
      D=K1(2)-K2(2)
      SLPAZ=(UAZ(1)-UAZ(2))/C
      SLPEL=(UEL(1)-UEL(2))/D
```

```
      DO 80 IP=1,LMAX
      LCTR=LCTR+1
      K(1)=(K1(1)+K2(1))*0.5
      K(2)=(K1(2)+K2(2))*0.5
      IF (ABS(C).GT.SLRAD) SLPAZ=(UAZ(1)-UAZ(2))/C
      IF (ABS(D).GT.SLRAD) SLPEL=(UEL(1)-UEL(2))/D
      BAZ=UAZ(1)-SLPAZ*K1(1)
      BEL=UEL(1)-SLPEL*K1(2)
      ACCAZ=ABS(SLPAZ*SCRAD+BAZ)
      ACCEL=ABS(SLPEL*SCRAD+BEL)
      K(1)=-BAZ/SLPAZ
      K(2)=-BEL/SLPEL
      K(3)=SQRT(1.-K(1)**2-K(2)**2)
      CALL INCPW(K,EINC,ICPT)
      CALL RECM(EINC,K,NY,NY,KXMAX,KYMAX,FGHZ,ROTATE,TRANSL,
     $ SUMX,SUMY,DELX,DELY,DAZX,DAZY,VR,TABLE,SUPPRS,RSQMAX)
      UA=AIMAG(VR(3)/VR(1))
      UE=AIMAG(VR(2)/VR(1))
      IF (IP.GT.1) GO TO 65
      IF (UA.GT.0.) GO TO 76
      UAZ(2)=UA
      K2(1)=K(1)
      JAZ=1
      GO TO 77
   76 UAZ(1)=UA
      K1(1)=K(1)
      JAZ=2
   77 IF(UE.GT.0.) GO TO 78
      UEL(2)=UE
      K2(2)=K(2)
      JEL=1
      GO TO 79
   78 UEL(1)=UE
      K1(2)=K(2)
      JEL=2
      GO TO 79
   65 IF (JAZ.EQ.2) GO TO 66
      UAZ(1)=UA
```

```
      K1(1)=K(1)
      JAZ=2
      GO TO 67
   66 UAZ(2)=UA
      K2(1)=K(1)
      JAZ=1
   67 IF (JEL.EQ.2) GO TO 68
      UEL(1)=UE
      K1(2)=K(2)
      JEL=2
      GO TO 79
   68 UEL(2)=UE
      K2(2)=K(2)
      JEL=1
   79 C=K1(1)-K2(1)
      D=K1(2)-K2(2)
      IF (IP.LT.3) GO TO 80
      IF ((ABS(UA).LT.ACCAZ).AND.(ABS(UE).LT.ACCEL)) GO TO 85
   80 CONTINUE
      WRITE(6,25)
   25 FORMAT(/" LMAX EXCEEDED BEFORE ACCURACY CRITERION MET"//)
   85 SMAX=CABS(VF(1))
      IF (ABS(C).GT.SDRAD) SLPAZ=(UAZ(1)-UAZ(2))/C
      IF (ABS(D).GT.SDRAD) SLPEL=(UEL(1)-UEL(2))/D
      AZTM=ASIN(K(1)/SQRT(1.-K(2)**2))*1000.
      ELTM=ASIN(K(2)/SQRT(1.-K(1)**2))*1000.
      K1(3)=SQRT(1.-K1(1)**2-K1(2)**2)
      K2(3)=SQRT(1.-K2(1)**2-K2(2)**2)
C  CONVERT SLOPES TO VOLTS/DEG, WHERE THE SIGNAL RECEIVED BY SUM
C  CHANNEL IS CONSIDERED TO BE ONE VOLT:
      SLPAZ=SLPAZ/57.3
      SLPEL=SLPEL/57.3
      IF (SUPPRS) RETURN
      WRITE(6,93) K1,K2,AZTM,ELTM,SLPAZ,SLPEL,UAZ,UEL,SMAX,LCTR
   93 FORMAT(//" FINAL ANSWERS FOR MONOPULSE SYSTEM: "//" K1: ",3E12.5/
     &" K2: ",3E12.5//" AZTM= ",E12.5," MRAD"/" ELTM= ",E12.5," MRAD"//
     & " MESAZ= ",E12.5," VOLTS/DEG"/" MESEL= ",E12.5," VOLTS/DEG"
     &//" UAZ: ",2E12.5/" UEL: ",2E12.5//" SMAX= ",E21.14,
```

```
  &"      LOTR= ",I3//)
   WRITE(6,98)
98 FORMAT(//" ADDITIONAL MONOPULSE OUTPUTS AROUND BORESIGHT: "//)
   DO 99 IP=1,7
   K1(1)=SIN((-3.+IP-1)/1000.)+K(1)
   K1(2)=SIN((-3.+IP-1)/1000.)+K(2)
   ANG=-3.+IP-1
   K1(3)=SQRT(1.-K1(1)**2-K1(2)**2)
   CALL INCPW(K1,EINC,IOPT)
   CALL RECM(EINC,K1,NX,NY,KXMAX,KYMAX,FGHZ,ROTATE,TRANSL,
  & SUMX,SUMY,DELX,DELY,DAZX,DAZY,VR,TABLE,SUPPRS,PSQMAX)
   UAZ(1)=AIMAG(VR(3)/VR(1))
   UEL(1)=AIMAG(VR(2)/VR(1))
   IF (IP.EQ.1) SLP1=UAZ(1)
   IF (IP.EQ.1) SLP2=UEL(1)
   WRITE(6,95) ANG,UAZ(1),UEL(1)
95 FORMAT(" ANG= ",F5.1," MRAD FROM BORESIGHT     VRAZ= ",E12.5,
  &" VOLTS     VREL= ",E12.5," VOLTS"/)
   VR(3)=VR(3)/VR(1)
   VR(2)=VR(2)/VR(1)
   CALL AMPHS(VR(3),C,D)
   CALL AMPHS(VR(2),E,F)
   WRITE(6,94) C,D,E,F
94 FORMAT(" DAZ(AMP,PHS)= ",2E12.5,"    DEL(AMP,PHS)= ",2E12.5/)
99 CONTINUE
   SLP1=(UAZ(1)-SLP1)/(.006*57.3)
   SLP2=(UEL(1)-SLP2)/(.006*57.3)
   WRITE(6,97) SLP1,SLP2
97 FORMAT(/" AVERAGE SLPAZ= ",E12.5," VOLTS/DEG"/
  &" AVERAGE SLPEL= ",E12.5," VOLTS/DEG"/"  SUM=1.0 VOLT"//)
100 RETURN
   END
```

Chapter 15

SUBROUTINE RECPTN

15-1. Purpose: To compute the receiving patterns of a monopulse antenna at NREC points in a specified principal plane. A plane wave of specified polarization (ICOMP) is made to be incident on the antenna at equal increments in $\sin\theta$ over the range (-KMAX, KMAX -DK) in either the elevation plane (ICUT = 1) or azimuth plane. The received voltage in each channel is computed in the presence of the radome and stored for return to the calling program.

15-2. Usage: CALL RECPTN (SUMX, SUMY, DELX, DELY, DAZX, DAZY,
NX, NY, ICUT, ICOMP, KMAX, NREC, VREC, KXMAX, KYMAX,
FGHZ, ROTATE, TRANSL, TABLE, SUPPRS, RSQMAX)

15-3. Arguments

SUMX, SUMY, DELX, DELY, DAZX, DAZY,	-	Complex input arrays of NX by NY elements containing the aperture field distributions of the monopulse antenna. See Subroutine HACNF.
ICUT	-	Integer input variable which specifies the principal plane in which the pattern is computed: elevation (ICUT = 1) or azimuth (ICUT = 2).
ICOMP	-	Integer input variable which specifies the linear polarization of the incident plane wave: elevation component $\hat{\epsilon}$ only (ICOMP = 1) or azimuth component $\hat{\alpha}$ only (ICOMP = 2). See Figure 15-1 for further clarification.

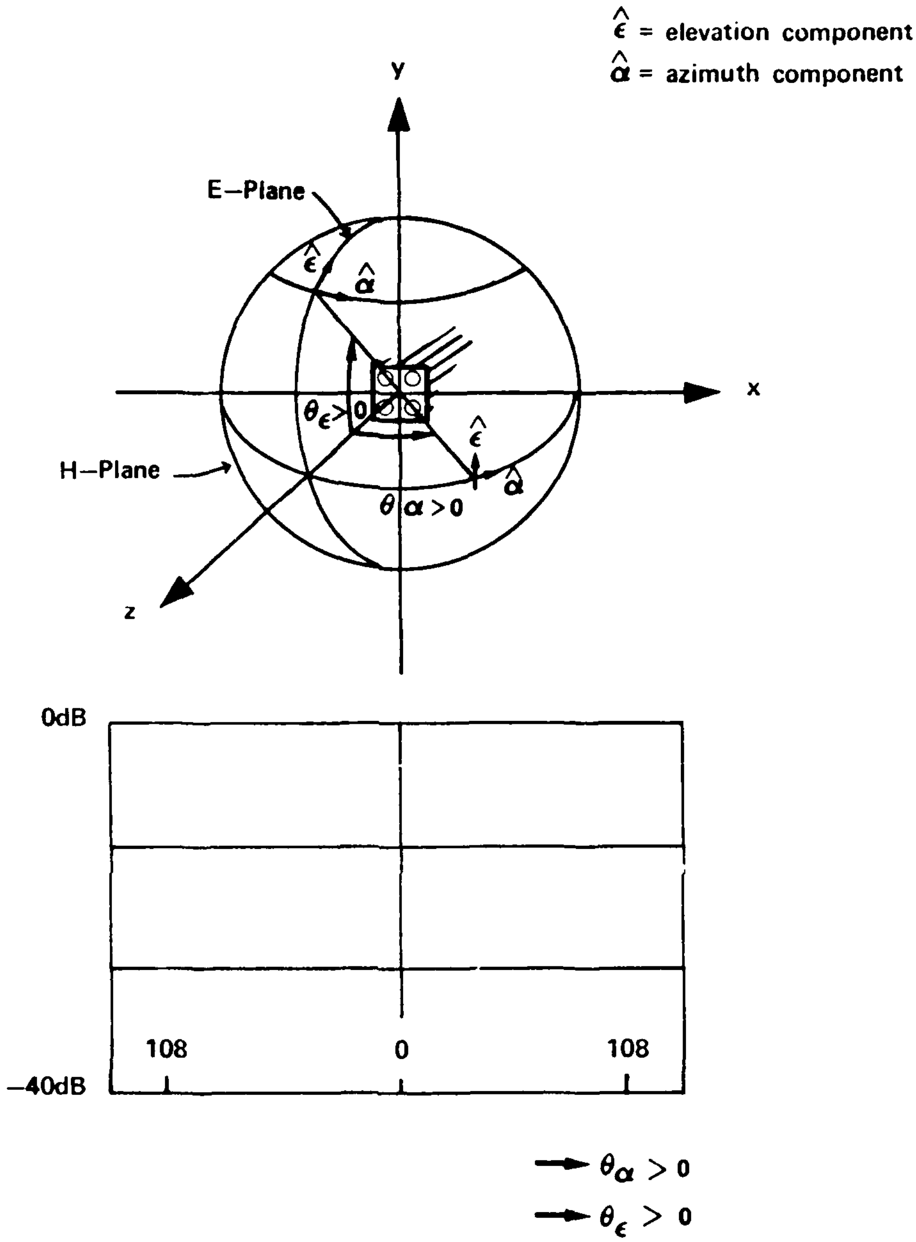

Figure 15-1 Coordinate System for Far Field Patterns

KMAX - Real input variable equal to $\sin\theta_{max}$, where the pattern is computed over the angular range ($-\theta_{max}$, θ_{max}), but in equal increments in $\sin\theta$ so that Fourier interpolation can be applied directly in the wavenumber domain using the Fast Fourier Transform.

NREC - Integer input variable equal to the number of points at which the pattern is computed.

VREC - Complex output array of NREC by 3 elements containing the computed receiving patterns for the sum, elevation difference, and azimuth difference patterns of the monopulse antenna.

KXMAX, KYMAX, FGHZ, ROTATE, TRANSL, TABLE, SUPPRS, RSQMAX - Input variables required by Subroutine RECM. See Chapter 8.

15-4. Comments and Method

Subroutines INCPW and RECM are used to compute the incident plane wave and the received voltage in each channel for each direction of arrival in the specified plane. For the elevation plane, the direction of arrival is given by

$$\hat{k} = \hat{x}\,(0) + \hat{y}\,\sin\theta + \hat{z}\sqrt{1-\sin^2\theta} \qquad (1)$$

where θ is the angle measured from the z-axis. For the azimuth plane

$$\hat{k} = \hat{x}\,\sin\theta + \hat{y}\,(0) + \hat{z}\sqrt{1-\sin^2\theta} \qquad (2)$$

The increments in angle are given by

$$\Delta k = 2\ k_{max}\ /\ N_{REC} \tag{3}$$

Values of $k_{max} > 1$ correspond to the invisible region and must be excluded from consideration.

15-5. Program Flow: Compare program listing below directly to the discussion above.

15-6. Test Case: See Chapter 2.

15-7. References: None

15-8. Program Listing: See following page.

```
      SUBROUTINE RECPTN(SUMX,SUMY,DELX,DELY,DAZX,DAZY,NX,NY,ICUT,ICOMP,
     & KMAX,NREC,VREC,KXMAX,KYMAX,FGHZ,ROTATE,TRANSL,
     * TABLE,SUPPRS,RSQMAX)
C SUBR RECPTN COMPUTES THE RECEIVING VOLTAGE PATTERN OF THE ANTENNA
C WHOSE TRANSMITTING NEAR FIELD ON ZA=0 PLANE IS EXT,EYT.
C RAY TRACING IS USED TO ACCOUNT FOR RADOME (SUBR RECM).
C NREC=NUMBER OF POINTS AT WHICH PATTERN IS COMPUTED
C ICUT=1 FOR EL CUT, =2 FOR AZ CUT
C ICOMP=1 FOR EL COMPONENT, =2 FOR AZ COMPONENT
C KMAX<1. SPECIFIES ANGULAR LIMITS (ANALAGOUS TO KXMAX)
      COMPLEX SUMX(NX,NY),SUMY(NX,NY),DELX(NX,NY),DELY(NX,NY)
      COMPLEX DAZX(NX,NY),DAZY(NX,NY)
      COMPLEX VREC(NREC,3),EINC(3),VR(3)
      REAL KXMAX,KYMAX,KMAX,KA(3),ROTATE(3,3),TRANSL(3)
      LOGICAL TABLE,SUPPRS
      DATA PI/3.14159265/
      FTR=0.120.*PI
      IF (IABS(ICUT).GT.2) ICUT=2
      J=1
      IF (KMAX.GE.1.) KMAX=1.-2./NREC
      DK=2.*KMAX/NREC
      IF (ICUT.EQ.1) J=2
      ANGMAX=ASIN(KMAX)*180./PI
      WRITE(6,10) ICUT,ICOMP,KMAX,NREC,DK,ANGMAX
      DO 5 I=1,NREC
      KA(ICUT)=0.
      KA(J)=-KMAX+(I-1)*DK
      KA(3)=SQRT(1.-KA(J)**2)
      CALL INCFW(KA,EINC,ICOMP)
      CALL RECM(EINC,KA,NX,NY,KXMAX,KYMAX,FGHZ,ROTATE,TRANSL,
     & SUMX,SUMY,DELX,DELY,DAZX,DAZY,VR,TABLE,SUPPRS,RSQMAX)
      VREC(I,1)=VR(1)
      VREC(I,2)=VR(2)
      VREC(I,3)=VR(3)
    5 CONTINUE
      WRITE(6,10) ICUT,ICOMP,KMAX,NREC,DK,ANGMAX
   10 FORMAT(/" RECEIVING PATTERN COMPUTED FOR: "/"    ICUT= ",I2/
     & "   ICOMP= ",I2/"    KMAX= ",F7.3/"    NREC= ",I5/"      DK= "
```

```
& ,I12.5/" ANGMAX= ",F6.2/" (ICUT=1 FOR EL CUT, =2 FOR AZ CUT"/
& " (ICOMP=1 FOR EL COMPONENT, =2 FOR AZIMUTH) "//)
  RETURN
  END
```

Chapter 16

SUBROUTINE OGIVE

16-1. Purpose: To solve for the intersection PH(x, y, z) of a line (ray) and a tangent ogive surface. The ray starts at point PO(x_o, y_o, z_o) and travels in the direction $K(k_x, k_y, k_z) = \hat{k} = \hat{x}k_x + \hat{y}k_y + \hat{z}k_z$. Dimensions are in centimeters. Radome coordinates are implied.

16-2. Usage: CALL OGIVE (PO, K, PH, HIT)

COMMON/OGIVC/RP, BSQ, AP, RINV, B, RSQ1, RP2

16-3. Arguments

PO - Real input array containing the point of origin of the ray PO(x_o, y_o, z_o).

K - Real input array containing the direction cosines of the ray K(k_x, k_y, k_z).

PH - Real output array containing the point of intersection PH(x, y, z), if HIT = .TRUE.

HIT - Logical output variable which indicates if an intersection solution was found (TRUE).

The following variables are common with the main program and are precalculated to speed up the ray tracing computations. Refer to Figure 16-1 of the radome geometry for the definitions of R and B.

RP - Real input variable = $R^2 - B^2$.

BSQ - Real input variable = B^2.

AP - Real input variable = 0. See APIN in Section 2-4.

RINV - Real input variable = 1/R.

B - Real input variable. See Figure 16-1.

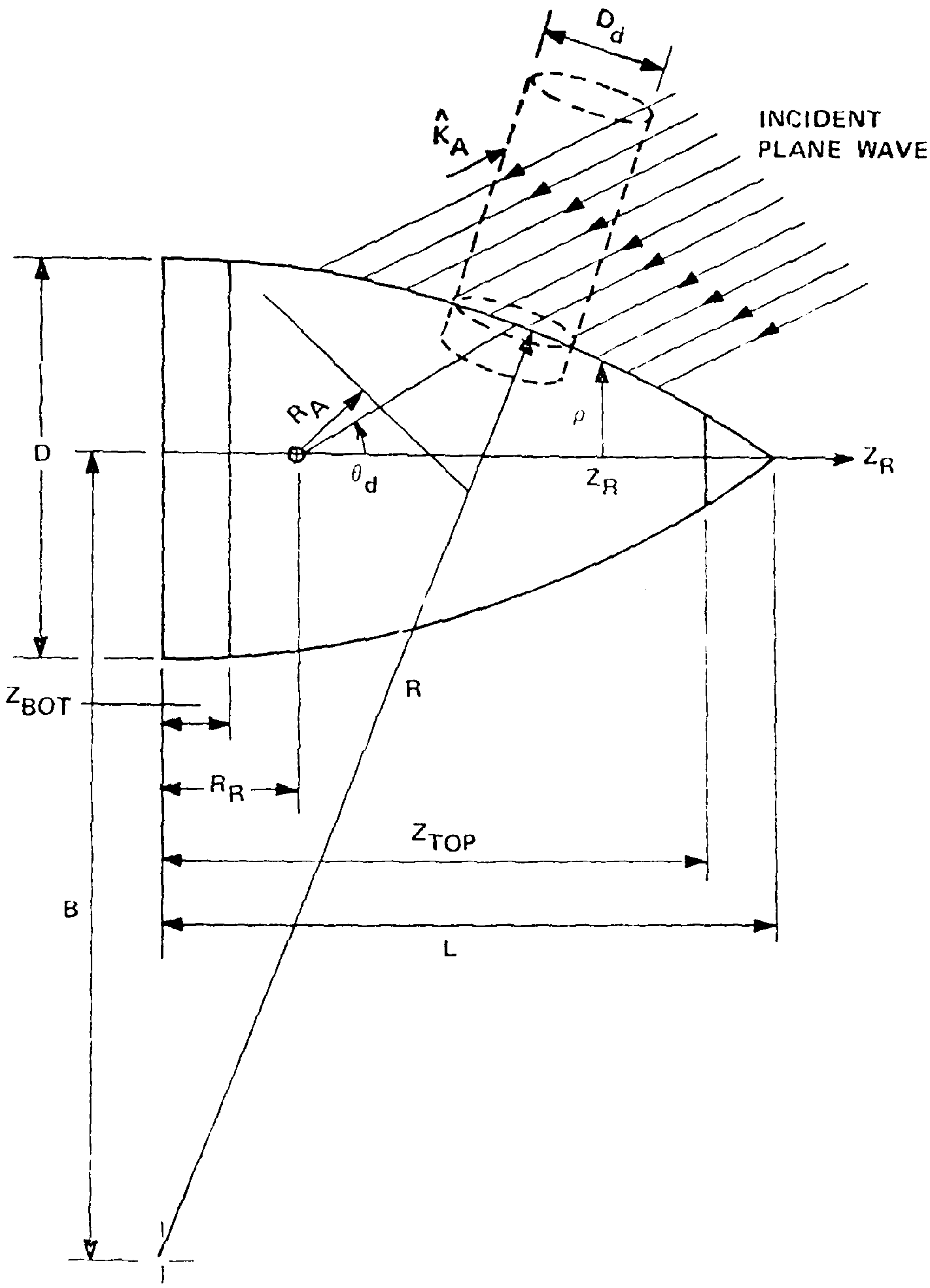

Figure 16-1. Tangent Ogive Radome Geometry.

RSQ1 - Real input variable = R^2.

RP2 - Real input variable = $R^2 + B^2$.

16-4. Comment and Method

a. The common variables must be computed in the main program prior to the first call to OGIVE.

b. Subroutines required: CBRT, SQR, XY. Real function CBRT(x) computes cube root; SQR computes square root with test for negative argument.

c. The intersection of a ray and ogive surface requires the solution of a quartic equation in the parameter z_p as follows [1]

$$z_p^4 + C_4 z_p^3 + C_3 z_p^2 + C_2 z_p + C_1 = 0 \tag{1}$$

where

$$C_4 = \frac{2(1+U)(2A+V)}{(1+U)^2} \tag{2}$$

$$C_3 = \frac{2(1+U)(-R_p+A^2+W) + (2A+V)^2 - 4B^2U}{(1+U)^2} \tag{3}$$

$$C_2 = \frac{2(2A+V)(-R_p+A^2+W) - 4B^2V}{(1+U)^2} \tag{4}$$

$$C_1 = \frac{(-R_p+A^2+W)^2 - 4B^2W}{(1+U)^2} \tag{5}$$

and where

$$U = \frac{K_1^2 + K_2^2}{K_3^2} \qquad (6)$$

$$V = \frac{2(K_1P_1 + K_2P_2)}{K_3} \qquad (7)$$

$$W = P_1^2 + P_2^2 \qquad (8)$$

$$K_3 = R^2 - B^2 \qquad (9)$$

The variables R and B are defined on Figure 16-1 and by the ogive equation

$$r = \sqrt{x^2 + y^2} = \sqrt{R^2 - (z-a_i)^2} - B \qquad (10)$$

where the z-axis is the axis of revolution for the ogive shape. The variable a_i provides for an offset along z of the coordinates for the ogive. Equations (6) through (9) result when the following equations for a ray which passes through the point $P0(x_o, y_o, z_o)$ in the direction $k = \hat{x}k_x + \hat{y}k_y + \hat{z}k_z$ are substituted into Equation (10):

$$\frac{z - z_o}{k_z} = \frac{x - x_o}{k_x} = \frac{y - y_o}{k_y} = \text{constant} \qquad (11)$$

All four roots of the quartic equation may be found from the re- [illegible]

[illegible] (12)

This cubic equation has at least one real root p_0 given by

$$\left[\frac{-T}{2} + \sqrt{\frac{T^2}{4} + \frac{S^3}{27}}\right]^{1/3} + \left[\frac{-T}{2} - \sqrt{\frac{T^2}{4} + \frac{S^3}{27}}\right]^{1/3} \qquad (13)$$

where

$$S = \frac{1}{3}[3(C_4C_2 - 4C_1) - C_3^2] \qquad (14)$$

$$T = \frac{1}{27}\,[-2C_3^3 + 9C_3(C_4C_2 - 4C_1) + 27(-C_4^2C_1 + 4C_3C_1 - C_2^2)] \qquad (15)$$

Once p_0 is found, the roots of the quartic equation follow from

$$z_{p1,2} = \frac{C_3}{4} + \frac{R_1}{2} \pm \frac{D}{2} \qquad (16)$$

$$z_{p3,4} = \frac{C_3}{4} - \frac{R_1}{2} \pm \frac{E}{2} \qquad (17)$$

where

$$R_1 = \sqrt{\frac{C_4^2}{4} - C_3 + p_0} \qquad (18)$$

$$D = \sqrt{\frac{3C_4^2}{4} - R_1^2 - 2C_3 + \frac{4C_4C_3 - 8C_2 - C_4^3}{4R_1}} \qquad (19)$$

$$E = \sqrt{\frac{3C_4^2}{4} - R_1^2 - 2C_3 - \frac{4C_4C_3 - 8C_2 - C_4^3}{4R_1}} \qquad (20)$$

The correct root z_{p0} is chosen as the one with the smallest absolute value and which has the same sign as k_z. The intersection point P(x, y, z)

follows that

$$z = z_0 + \gamma_{po} \qquad ([illegible])$$

$$x = \frac{k_x}{k_z} \, [illegible]_{po} + x_0 \qquad ([illegible])$$

$$y = \frac{k_y}{k_z} \, [illegible]_{po} + y_0 \qquad ([illegible])$$

The rectangular components of the unit inward normal vector at P(x, y, z) are given by

$$n_x = -x \, \frac{R + \sqrt{x^2+y^2}}{R\sqrt{x^2+y^2}} \qquad (24)$$

$$n_y = -y \, \frac{R + \sqrt{x^2+y^2}}{R\sqrt{x^2+y^2}} \qquad ([illegible])$$

$$n_z = -\frac{z - a}{R} \qquad (26)$$

In the the special case of $k_z = 0$, the z coordinate does not change so that

$$z = z_0 \qquad (27)$$

The equations of the line in the $z = z_0$ plane become

$$x = x_0 + k_x t \qquad (28)$$

$$y = y_o + k_y t \tag{29}$$

where the parameter t is the distance along the line from (x_o, y_o, z_o) to (x, y, z). Substituting Equations (27) - (29) into (10) yields the following quadratic equation in t

$$t^2 + 2(k_x x_o + k_y y_o)t + (x_o^2 + y_o^2) - (\sqrt{R^2 - (z_o - a_1)^2} - B)^2 = R_s^2 \tag{30}$$

The quadratic formula yields the following solutions to the above equation

$$t_{1,2} = \text{[illegible]} (k_x x_o + k_y y_o)^2 - (x_o + y_o^2) + R_s^2]^{\frac{1}{2}}$$

The [illegible] Equations (24) through (26).

1 - [illegible] and compare directly to the above [illegible]

1 [illegible]

1 - [illegible]

1. [illegible], "Radome Effects on the [illegible] Radar," Technical Report, Contract [illegible], U. S. Army Missile Command, March 197[illegible].

2. Stegun, and Abromowitz, <u>Handbook of Mathematical Functions</u>, National Bureau of Standards, June 1964, p. 17.

10-8. Program Listing: See following pages.

```
      SUBROUTINE OGIVE(P0,K,PH,HIT)
      REAL P0,K,M1,M2,U,V,W,ABS,AINV,A0,A1,A2,A22,A44,COEFF(
     4)
C THIS SUBROUTINE SOLVES FOR THE INTERSECTION OF A RAY AND AN OGIVE.
C        INPUT   P0---REAL ARRAY OF COEFFICIENTS OF INSIDE POINT
C                K ---REALARRAY OF DIRECTION COSINES OF RAY
C        OUTPUT  PH---COORDINATES OF THE INTERSECTION WITH THE RADOME
C                HIT---=.TRUE. IF THE RAY HIT THE SURFACE
      DIMENSION P0(3),PH(3),K(3)
      COMMON/OGIV/RP,B2C,AP,PINV,B,RSQ1,RP2
      LOGICAL HIT
      HIT=.TRUE.
C UNIT OF DISTANCE IS CENTIMETER
C THE OGIVE SHAPE IS DESCRIBED BY THE EQUATION
C       SQRT(X**2+Y**2)=SQRT(R**2-(Z-AP)**2)-B
C GIVEN A FINENESS RATIO ,F, AND A DIAMETER D, R AND B
C MAY BE CALCULATED AS FOLLOWS.
C  R=F*D/(SIN(PI-2*ATAN(2*F))
C  B=R-D/2
C AP= Z COORDINATE OF THE GENERATION CENTER OF THE OGIVE
C       COORDINATES.
C THE DATA NEEDED FOR THE DATA STATEMENT IS
C            RP=R**2-B**2
C            B2C=B**2
C            AP=AP
C            PINV=1./R
C            B=B
C            RSQ1=R**2
C            RP2=R**2+B**2
      IF(ABS(K(3)).LT.1E-7)GO TO 330
      IF(ABS(K(3)).GT.1.-1E-7) GO TO 340
      A=P0(3)-AP
      M1=1./K(3)
      M2=K(2)*M1
      M1=K(1)*M1
      U=M1*M1+M2*M2
      V=2.*(M1*P0(1)+M2*P0(2))
      W=P0(1)*P0(1)+P0(2)*P0(2)
```

```
      ASQ=A*A
      AINV=1.0/(1.0+U*(2.0+U))
      COEF (4)=2.0*(2.0*A+V)*(1.0+U)*AINV
      COEF(3)=(2.0*(-RP+U*(W-RP2          )+W+2.0*A*V+ASQ*(3.0+U))+V*V)*AINV
      COEF(2)=(4.0*A*(-RP+ASQ+W)+2.0*V*(ASQ+W-RP2          ))*AINV
      COEF(1)=(W*(2.0*(ASQ-RP2          )+W)+RP*(RP-2.0*ASQ)+ASQ*ASQ)*AINV
      A2=COEF(3)*0.3333333333
      A1=COEF(2)*COEF(4)-4.0*COEF(1)
      A0=-COEF(3)*COEF(2)-COEF(1)*COEF(4)*COEF(4)+4.0*COEF(1)*COEF(3)
      A22=A2*A2
      A44=COEF(4)*0.25
      Q=A1*.3333333333-A22
      R1=(-A1*COEF(3)-3.0*A0)*0.1666666667+A2*A22
      RADSQ=Q**3+R1*R1
      IF (RADSQ.LT.0.) GO TO 330
      RAD=SQR(RADSQ)
      Y=CBRT(R1+RAD)+CBRT(R1-RAD)+A2
  340 RSQ=COEF(4)*A44-COEF(3)+Y
      IF (ABS(RSQ).LT.1.0E-05) GO TO 350
      R=SQR(RSQ)
      IF (K(3).LT.0.) GO TO 360
      Z=PC(3)-A44+(R-SQR(3.0*COEF(4)*A44-RSQ-2.0*COEF(3)+(4.0*COEF(4)*
     2          COEF(3)-8.0*COEF(2)-COEF(4)**3)/(4.0*R)))*0.5
         CALL XY(PC,K,Z,PH)
      RETURN
  360 Z=PC(3)-A44-(R-SQR(3.0*COEF(4)*A44-RSQ-2.0*COEF(3)-(4.0*COEF(4)*
     2          COEF(3)-8.0*COEF(2)-COEF(4)**3)/(4.0*R)))*0.5
         CALL XY(PC,K,Z,PH)
      RETURN
  330 PHI =ACOS(R1/SQR(-Q**3))
      Y=2.0*SQR(-Q)*COS(PHI*0.3333333333)+A2
      GO TO 340
  350 IF (K(3).LT.0.0) GO TO 370
      Z=PC(3)-A44+(R-SQR(3.0*COEF(4)*A44-2.0*COEF(3)+2.0*SQR(Y*Y-
     2       4.0*COEF(1))))*0.5
         CALL XY(PC,K,Z,PH)
      RETURN
  370 Z=PC(3)-A44+(R+SQR(3.0*COEF(4)*A44-2.0*COEF(3)-2.0*SQR(Y*Y-
```

```
      2     +..*SQRT(1)))*0.5
            CALL  TY(PO,K,Z,PH)
            RETURN
  380   Z=H+SIGN(1.,K(3))*SQRT(RSQ1-(SQRT( (PO(1))**2+ (PO(2))**2)+
     *  (H))**2)
        PH(1)=PO(1)
        PH(2)=PO(2)
        PH(3)=Z
        RETURN
  390   RSQ2=SQRT(RSQ1-(PO(3)-AF)**2)-R
        RSQ2=RSQ2**2
        PLP=-(PO(1)*K(1)+PO(2)*K(2))+SQRT((PO(1)*K(1)+PO(2)*K(2))**2-
     *(PO(1)**2+PO(2)**2-RSQ2))
        RLN=-(PO(1)*K(1)+PO(2)*K(2))-SQRT((PO(1)*K(1)+PO(2)*K(2))**2-
     *(PO(1)**2+PO(2)**2-RSQ2))
        PH(3)=PO(3)
        IF(RLN.LT.0.) GO TO 391
        PH(1)=PO(1)+RLN*K(1)
        PH(2)=PO(2)+RLN*K(2)
        RETURN
  391   PH(1)=PO(1)+PLP*K(1)
        PH(2)=PO(2)+PLP*K(2)
        RETURN
        END
```

```
FUNCTION CBRT(X)
SIGNX=1.0
IF(X.LT.0.0) SIGNX=-1.0
CBRT=SIGNX*(SIGNX*X)**0.3333333333333
RETURN
END
```

```
      FUNCTION SQH(SQ)
      IF(SQ.LE.0.)GO TO 49
      SQH=SQRT(SQ)
      RETURN
49    SQH=0.
      RETURN
      END
```

Chapter 17

SUBROUTINE OGIVEN

17-1. Purpose: To compute the unit inward normal vector $\hat{n} = \hat{x}\, n_x + \hat{y}\, n_y + \hat{z}\, n_z$ to the tangent ogive surface at the point PI(x, y, z). Dimensions are in centimeters and radome coordinates are implied.

17-2. Usage: CALL OGIVEN (PI, N)

COMMON/OGIVC/RP, BSQ, AP, RINV, B, RSQ1, RP2

(See Chapter 16 for common variables.)

17-3. Arguments

PI - Real input array containing the point PI(x, y, z) on the tangent ogive surface at which the unit normal is desired, as computed by Subroutine OGIVE.

N - Real output array containing the direction cosines (n_x, n_y, n_z) of the unit inward normal vector.

17-4. Comments and Method

The tangent ogive surface is described by

$$f(r,z) = r - \sqrt{R^2 - (z-a_i)^2} + B = 0 \tag{1}$$

where $r = \sqrt{x^2+y^2}$ and where R and B are defined in Figure 16-1. The unit inward normal to this surface is given by

$$\hat{n} = -\frac{\nabla f}{|\nabla f|} = -\frac{1}{|\nabla f|}\left[\hat{x}\,\frac{df}{dr}\frac{dr}{dx} + \hat{y}\,\frac{df}{dr}\frac{dr}{dy} + \hat{z}\,\frac{df}{dz}\right] \tag{2}$$

where ∇ is the gradient operator. Equation (2) can be rewritten as

$$\hat{n} = \frac{-1}{|\nabla f|}\left[\hat{x}\,\frac{x}{r} + \hat{y}\,\frac{y}{r} + \hat{z}\,\frac{df}{dz}\right] \tag{3}$$

where the differentiation with respect to r has been done and df/dr = 1 has been used. The remaining terms are given by

$$\frac{df}{dz} = \frac{(z-a_1)}{\sqrt{R^2 - (z-a_1)^2}} \tag{4}$$

$$|\nabla f| = \sqrt{1 + \frac{(z-a_1)^2}{R^2 - (z-a_1)^2}} = \frac{R}{\sqrt{R^2 - (z-a_1)^2}} \tag{5}$$

where $r^2 = x^2 + y^2$. The direction cosines can be written explicitly as

$$n_z = -(z-a_1)/R \tag{6}$$

$$n_x = -\left(\frac{x}{r}\right)\frac{\sqrt{R^2 - (z-a_1)^2}}{R} = -\frac{x}{r}\,\frac{(r + B)}{R} \tag{7}$$

$$n_y = -\left(\frac{y}{r}\right)\frac{(r+B)}{R} \tag{8}$$

where the relation $(r+B) = \sqrt{R^2 - (z-a_1)^2}$ from Equation (1) has been used.

[illegible] Flow: [illegible] Equations (6) - (8) directly to the [illegible] flow.

[illegible] See Chapter [illegible].

17-7. Reference

1. Smail, L. L., Analytic Geometry and Calculus, Appleton-Century-Crofts, Inc., New York, 1953.

17-8. Program Listing: See following page.

```
S[illegible] [illegible]IV[illegible] [illegible])
[illegible] [illegible]ALCULA[illegible] TH[illegible] I[illegible]WAR[illegible] [illegible]ORMAL TO TH[illegible] OGIVE SURFACE
[illegible] [illegible]
 [illegible] [illegible]-[illegible]AL ARRAY [illegible] [illegible]EFFICI[illegible]NTS OF THE POINT
 [illegible] [illegible]--[illegible]AL ARRAY [illegible] [illegible]CTION [illegible]SINES OF NORMAL TO
 [illegible] AT POINT OF INT[illegible]ION
[illegible] [illegible](3),[illegible],[illegible]I(3)
[illegible]/[illegible],[illegible],[illegible]-I[illegible]V,[illegible]
[illegible](3)=-(PI([illegible])-[illegible])*[illegible]INV
[illegible]=[illegible](PI(1)*PI(1)+PI(2)*PI(2))
[illegible]=-([illegible]+[illegible])*[illegible]
[illegible](1)=[illegible]I(1)*[illegible]TA[illegible]
[illegible](2)=PI(2)*[illegible]TA[illegible]
R[illegible]
E[illegible]
```

Chapter 18

SUBROUTINE XY

18-1. Purpose: To compute the x and y coordinates of the intersection point PI(x, y, z) of a line (ray) having direction cosines $K(k_x, k_y, k_z)$ with a surface of revolution when z is known. The line passes through the known point $P(x_o, y_o, z_o)$. All dimensions are in centimeters.

18-2. Usage: CALL XY (P, K, Z, PI)

18-3. Arguments

P - Real input array containing the known point through which the ray passes; i.e., $P(x_o, y_o, z_o)$.

K - Real input array of the direction cosines of the ray; i.e., $K(k_x, k_y, k_z)$.

Z - Real input variable equal to the known z coordinate of the intersection as found, for example, from Subroutine OGIVE.

PI - Real output array containing the desired point of intersection PI(x, y, z).

18-4. Comments and Method

The parametric equations for a line in space passing through the point $P(x_o, y_o, z_o)$ and having direction cosines (k_x, k_y, k_z) are given by

$$x = x_o + k_x t \tag{1a}$$

$$y = y_o + k_y t \tag{1b}$$

$$z = z_o + k_z t \tag{1c}$$

where [illegible] is the distance along the line from I(x_0, y_0, z_0) to II(x, y, z). [illegible] coordinates [illegible] known, it follows from Equation ([illegible]), provided [illegible].

[illegible] Laminar Flow: [illegible] the [illegible] [illegible] V.

[illegible] Chapter 2.

[illegible]-7. [illegible] No.

[illegible] following [illegible].

```
      SUBROUTINE XY(P,K,Z,PI)

XY CALCULATES THE X AND Y COMPONENTS OF AN INTERSECTION POINT PI
FOR THE CASE WHEN THE POINT OF ORIGINATION P, THE DIRECTION OF
PROPAGATION K AND THE Z COORDINATE OF THE INTERSECTION POINT
ARE GIVEN

      REAL P(3),K(3),PI(3)
      PI(3)=Z
      T=(PI(3)-P(3))/K(3)
      PI(1)=P(1)+K(1)*T
      PI(2)=P(2)+K(2)*T
      RETURN
      END
```

Chapter 19

SUBROUTINES BDISK, BDISKN, TDISK, TDISKN

19-1. Purpose: To compute the intersection PI(x, y, z) of a line (ray) emanating from the point $P(x_o, y_o, z_o)$ having direction cosines $K(k_x, k_y, k_z)$ with a planar disk at $z = z_{bot}$ or at $z = z_{top}$. Subroutine BDISKN is used to compute the unit inward normal $\hat{n} = \hat{z}$. Subroutine TDISKN is used to compute the normal $\hat{n} = -\hat{z}$.

19-2. Usage:

```
CALL BDISK (P, K, PI, HIT)        CALL TDISK (P, K, PI, HIT)
COMMON/BDISKC/ZBOT, RBSQ          COMMON/TDISKC/ZTOP, RTSQ
CALL BDISKN (N)                   CALL TDISKN (N)
```

19-3. Arguments

- P - Real input array containing the point $P(x_o, y_o, z_o)$ from which the ray emanates.
- K - Real input array of direction cosines $K(k_x, k_y, k_z)$.
- PI - Real input array containing the desired point of intersection PI(x, y, z).
- HIT - Logical output variable which is TRUE if an intersection is found.
- ZBOT - Real input variable equal to the z coordinate of the planar disk.
- RBSQ - Real input variable equal to the square of the radius of the planar disk.
- N - Real output array containing the direction cosines of the unit inward normal vector; viz., N(0, 0, 1).

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1-4. Comments and Method

From the parametric equations for the ray

$$x = x_o + k_x t \tag{1a}$$

$$y = y_o + k_y t \tag{1b}$$

$$z = z_o + k_z t \tag{1c}$$

and the equation of the plane $z = z_{bot}$, the parameter t is given by

$$t = (z_{bot} - z_o)/k_z \tag{2}$$

provided $k_z \neq 0$. The x and y coordinates follow from the above equations; however, if $(x^2 + y^2) > r_b^2$ (where r_b is the radius of the disk), no intersection is found. Similar statements apply for the top disk.

1-5. Program Flow: Compare the listings below directly to the equations above.

1-6. Test Case: See Chapter 2.

1-7. References: None

1-8. Program Listing: See following pages.

```
      SUBROUTINE BDISK(P,K,PI,HIT)
C
C     WHICH CALCULATES THE POINT OF INTERSECTION PI OF A DISK HORIZONTAL
C     TO THE XY PLANE WITH A RAY EMANATING FROM POINT P AND TRAVELING
C     IN THE K DIRECTION.
C
C      THE EQUATION USED FOR THE BOT DISK IS Z=ZBOT  FOR (X**2+Y**2)<RSQ
C
      COMMON/BDISK/ZBOT,RBSQ
      REAL P(3),K(3),PI(3)
      LOGICAL HIT
      ZT=ZBOT-P(3)
      IF(ZT.GE.0.0) GO TO 1
      PI(3)=ZBOT
      T=ZT/K(3)
      PI(1)=P(1)+K(1)*T
      PI(2)=P(2)+K(2)*T
      PITSQ=PI(1)*PI(1)+PI(2)*PI(2)
      IF(PITSQ.GT.RBSQ+0.1) GO TO 1
      HIT=.TRUE.
      RETURN
    1 HIT=.FALSE.
      RETURN
      END
```

```
      SUBROUTINE [illegible]TRN(N)
C
C [illegible] UNIT OUTWARD NORMAL OF BOTTOM DISK
C
      REAL N(3)
      N(1)=0.0
      N(2)=0.0
      N(3)=-1.0
      RETURN
      END
```

```
      SUBROUTINE TDISK(P,K,PI,HIT)
C
C     THIS CALCULATES THE POINT OF INTERSECTION PI OF A DISK HORIZONTAL
C     TO THE XY PLANE WITH A RAY EMANATING FROM POINT P AND TRAVELING
C     IN THE K DIRECTION.
C
C       THE EQUATION USED FOR THE TOP DISK IS Z=ZTOP  FOR (X**2+Y**2)<RSQ
C
      COMMON/TDISKC/ZTOP,RTSQ
      REAL P(3),K(3),PI(3)
      LOGICAL HIT
      ZT=ZTOP-P(3)
      IF(ZT.LE.0.0) GO TO 1
      PI(3)=ZTOP
      T=ZT/K(3)
      PI(1)=P(1)+K(1)*T
      PI(2)=P(2)+K(2)*T
      PITSQ=PI(1)*PI(1)+PI(2)*PI(2)
      IF(PITSQ.GT.RTSQ+0.1) GO TO 1
      HIT=.TRUE.
      RETURN
    1 HIT=.FALSE.
      RETURN
      END
```

```
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
```

Chapter 20

SUBROUTINE FAR

20-1. Purpose: To compute the far field pattern in wavenumber coordinates (k_x, k_y) of an antenna whose radiating characteristics are specified by the complex plane wave spectra $A_x(k_x, k_y)$, $A_y(k_x, k_y)$. The antenna is located in a plane perpendicular to the z (polar) axis.

20-2. Usage: CALL FAR (FIELD, XFIELD, YFIELD, NX, NY, FGHZ, KXMAX, KYMAX, RADIUS, IPWR, FMAX)

20-3. Arguments:

FIELD - Real output array of NX by NY elements containing the far field power pattern at discrete wavenumbers $k_x = \sin\theta\cos\phi$, $k_y = \sin\theta\sin\phi$, where θ and ϕ are the usual polar and azimuthal angles.

XFIELD, YFIELD - Complex input arrays of NX by NY elements containing the plane wave spectra A_x, A_y at discrete wavenumbers k_x, k_y.

NX, NY - Integer input variables equal to the array sizes.

FGHZ - Real input variable equal to the frequency in gigahertz.

KXMAX, KYMAX - Real input variables equal to the maximum wavenumber associated with the elements of the arrays FIELD, XFIELD, and YFIELD. The element I=1, J=1 in these arrays corresponds to the wavenumber coordinate (-KXMAX, -KYMAX). For any (I,J), the wavenumber coordinates are given by

$$KX = (I - \frac{NX}{2} - 1) * KXINC$$

$$KY = (J - \frac{NY}{2} - 1) * KYINC$$

where

$$KXINC = 2*KXMAX/NX$$

$$KYINC = 2*KYMAX/NY$$

RADIUS - Real input variable equal to the radius r in centimeters of the sphere on which the far field pattern is computed. This variable effects only the term e^{-jkr}/r, and r is set to unity in the calling program for normal use.

IPWR - Integer input variable which selects the vector components to be used in computing the power pattern:

1 = Elevation component only

2 = Azimuth component only

3 = Total power

4 = Right hand circular polarization

Left hand circular polarization

FMAX - Real input and output variable. On input, if FMAX $\leq$ 0, the program will normalize the array FIELD from zero to one and output the normalizing factor as FMAX. If FMAX > 0 on input, it will be used as the normalizing factor; on output it will be unchanged.

20-4. Comments and Method

Let $E_x(x, y, 0)$, $E_y(x, y, 0)$ be the tangential electric fields of a rectangular antenna aperture located in the z = 0 plane and centered at the origin of the coordinate system. The plane wave spectra of the aperture fields are defined by

$$A_x(k_x, k_y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_x(x, y, 0)\, e^{+j(k_x x + k_y y)}\, dxdy \qquad (1)$$

$$A_y(k_x, k_y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_y(x, y, 0)\, e^{+j(k_x x + k_y y)}\, dxdy \qquad (2)$$

$$A_z(k_x, k_y) = \frac{-k_x A_x - k_y A_y}{k_z} \qquad (3)$$

where

$$k_x^2 + k_y^2 + k_z^2 = k^2 = \left(\frac{2\pi}{\lambda}\right)^2 \qquad (4)$$

The electric field E_ξ (ξ = x, y, or z) at any point (x, y, z > 0) is given by

$$E_\xi(x, y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A_\xi(k_x, k_y)\, e^{-j\, \underline{k}\cdot\underline{r}}\, dk_x dk_y \qquad (5)$$

where

$$\underline{r} = \hat{x}x + \hat{y}y + \hat{z}z \qquad (6)$$

$$\underline{k} = \hat{x}k_x + \hat{y}k_y + \hat{z}k_z \qquad (7)$$

And for the special case of large r, the rectangular field components E_i approach their asymptotic values [1]

$$E_{iff}(r, k_{xo}, k_{yo}) \sim j2\pi k \frac{e^{-jkr}}{r} \cos\theta \, A_i(k_{xo}, k_{yo}) \tag{8}$$

where the stationary phase points are given by

$$k_{xo} = k \sin\theta \cos\phi \tag{9}$$

$$k_{yo} = k \sin\theta \sin\phi \tag{10}$$

$$k_{zo} = k \cos\theta \tag{11}$$

In the above equations, θ is the polar angle measured from the z axis, and ϕ is the azimuthal angle measured from +x toward +y in the conventional spherical coordinate manner.

Consider the antenna measurement coordinate system in Figure [illegible]-1. Let the wavenumbers k_x, k_y, k_z be normalized by k [illegible], so that for $k_x^2 + k_y^2$ [illegible] 1, they represent direction cosines of the direction specified by [illegible], or equivalently by [illegible]. In terms of these normalized wavenumbers, the unit vectors [illegible] may be written as

$$[illegible] = \hat{x} \frac{-k_x k_y}{\sqrt{1 - k_y^2}} + \hat{y}\sqrt{1-k_y^2} + \hat{z} \frac{-k_y k_z}{\sqrt{1 - k_y^2}} \tag{12}$$

$$[illegible] = \hat{x} \frac{k_z}{\sqrt{1 - k_y^2}} + \hat{y}(0) + \hat{z} \frac{-k_x}{\sqrt{1 - k_y^2}} \tag{13}$$

[illegible] components [illegible] the far field E_{ff} then follow [illegible]

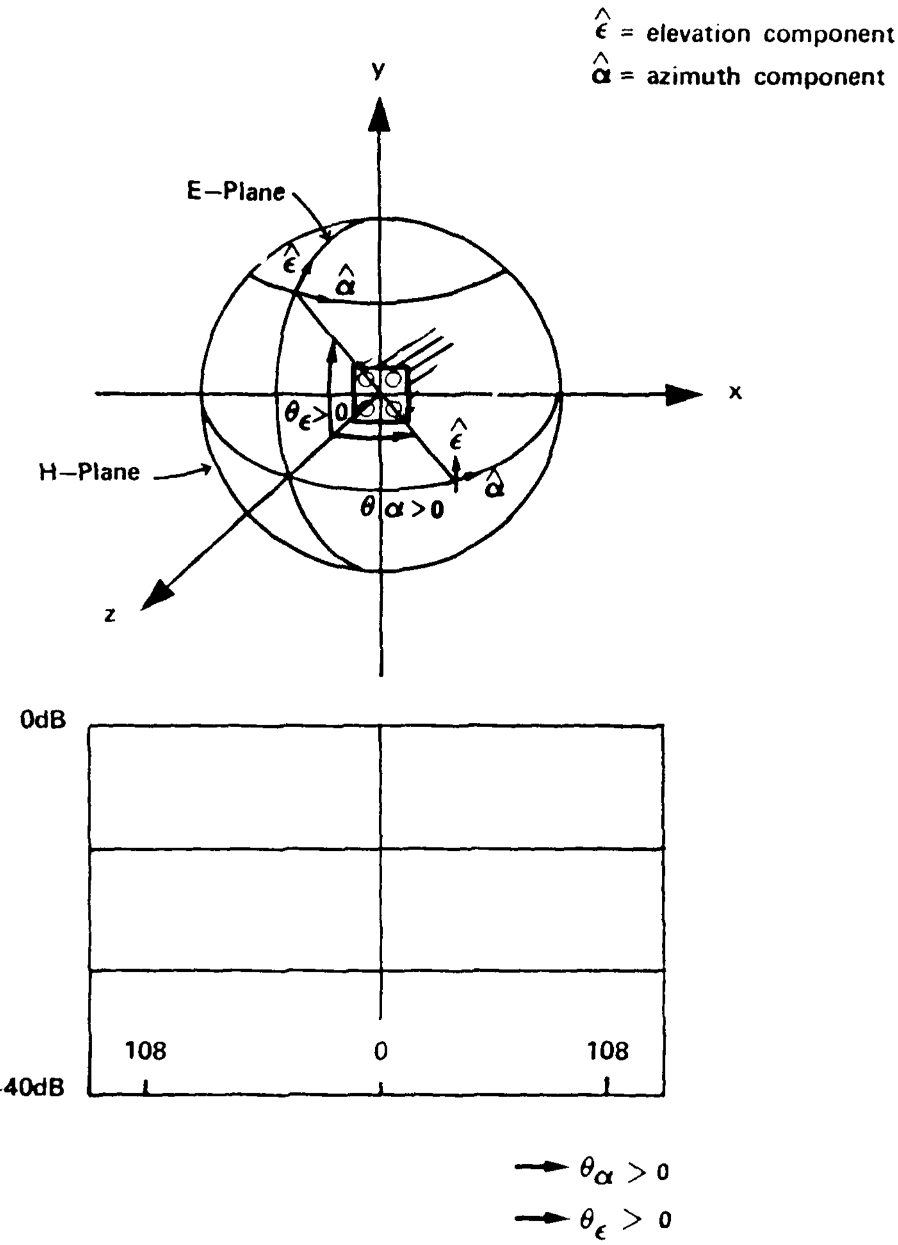

Figure -1 Coordinate System for Far Field Patterns

$$E_{ff} \; [illegible] \; \underline{E}_{ff} \; [illegible] \; j2\pi k \frac{e^{-jkr}}{r} \; [illegible] \cdot [\hat{x}\, k_z A_x + \hat{y}\, A_y + \hat{z}(-k_x A_x - k_y A_y)] \quad (14)$$

$$E_{\phi ff} \; [illegible] \; \frac{e^{-jkr}}{r} \; [illegible] \cdot E_{ff} \quad (15)$$

(In Subroutine FAR, the factor $j2\pi k$ is not used, and the plane wave spectra A_x, A_y are provided as computed previously using the Fast Fourier Transform.)

It is convenient to recall that the receiving and transmitting patterns of an antenna are identical, and that the receiving pattern $V_R(k)$ is given in terms of the far field $\underline{E}_{ff}$ by [2]

$$V_R(k) = C\, \hat{n}_i \cdot \underline{E}_{ff}(k) \quad (16)$$

where $\hat{n}_i$ [illegible] current source (probe) located on the far-field [illegible] constant which is set to unity for [illegible] receiving [illegible] pattern is [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

and is the receiving pattern when the probe is polarization matched at every point to the test antenna.

In the case of circularly polarized fields, the probe $\hat{n}_b$ can be expressed as

$$\text{RHC:} \quad \hat{n}_b = \frac{\hat{\theta} + \hat{\phi}\, e^{-j\frac{\pi}{2}}}{\sqrt{2}} \tag{20}$$

$$\text{LHC:} \quad \hat{n}_b = \frac{\hat{\theta}\, e^{-j\frac{\pi}{2}} + \hat{\phi}}{\sqrt{2}} \tag{21}$$

The receiving patterns in the two cases are then given by

$$|V_R|^2_{\_HC} = |\hat{n}_b \cdot \underline{E}_{ff}|^2 \tag{22}$$

where the appropriate $\hat{n}_b$ is used.

Subroutine FAR implements the above equations and computes the power patterns for an aperture in an infinite ground plane; i.e., the use of only A_x and A_y is tantamount to the assumption that $\underline{E}_{tan}$ outside the finite aperture area is zero. For the extended case of a finite aperture in free space, the tangential magnetic field $\underline{H}_{tan}$ also contributes to the radiated field, and the far fields are given by Equations (3-46) - (3-49) of Reference 5. In fact, it is only by including the effects of $\underline{H}_{tan}$ that the transmitting and receiving formulations for the finite aperture can be shown to be equivalent [4].

The current version of Subroutine FAR listed below could be easily modified to include the additional terms. If the geometrical optics approximation for the aperture fields is made; viz.,

$$\underline{H}_{tan} = \frac{\hat{z} \times \underline{E}_{tan}}{\eta} \qquad (23)$$

then the far-field expressions become

$$\begin{aligned} \underline{E}_{ff}(k_x, k_y) = \; & \hat{x}[(k_z + 1 - k_x^2) A_x - k_x k_y A_y] \\ & + \hat{y}[-k_x k_y A_x + (k_z + 1 - k_y^2) A_y] \\ & + \hat{z}[-k_x(1 + k_z) A_x - k_y(1 + k_z) A_y] \end{aligned} \qquad (24)$$

These modifications would involve changes only to Lines 7[illegible] - 7[illegible] of Subroutine FAF.

[illegible]-5. Program Flow: Compare listing below directly to Equations (1) - (22) above.

[illegible]-6. Test Case: See Chapter [illegible].

[illegible]-7. References

1. P. C. Clemmow, The Plane Wave Spectrum Representation of Electromagnetic Fields, Pergamon Press, Oxford, 1966.

2. G. K. Huddleston, H. L. Bassett, and J. M. Newton, "Parametric Investigation of Radome Analysis Methods", 1978 IEEE/AP-S Symposium Digest, pp. [illegible], May 1978, and in Proc. of the Fourteenth Symposium on Electromagnetic Windows, pp. [illegible], June 1978.

3. [illegible] F. [illegible], "Spectrum Probes for Near-Field Measurements [illegible]", Ph.D. Dissertation, Georgia Institute of Technology, Atlanta, Georgia, August 197[illegible].

4. [illegible] K. Huddleston, "[illegible] of [illegible] and [illegible] in Radome Analysis", [illegible].

[illegible]

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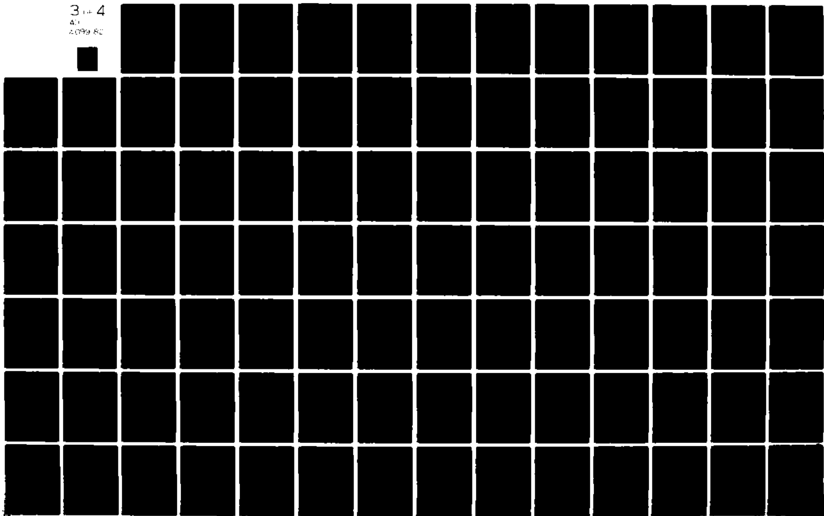

```
      SUBROUTINE FAR(FIELD,XFIELD,YFIELD,NX,NY,FGHZ,KXMAX,KYMAX,
     $ RADIUS,IPWR,FMAX)
C
C     FIELD IS A TWO DIMENSIONAL REAL ARRAY (NX,NY).  ON OUTPUT
C         IT CONTAINS THE FAR FIELD POWER PATTERN OF A COMPLEX
C         VECTOR PLANE WAVE SPECTRUM.
C     XFIELD AND YFIELD ARE TWO DIMENSIONAL COMPLEX ARRAYS WHICH
C         CONTAIN RESPECTIVELY THE X AND Y COMPONENTS OF A
C         COMPLEX PLANE WAVE SPECTRUM
C     KXMAX AND KYMAX ARE RESPECTIVELY THE MAXIMUM ABSOLUTE
C         VALUES OF KX AND KY WAVENUMBERS FOR WHICH THE FAR FIELD
C         IS CALCULATED.  KXMAX AND KYMAX ARE NORMALIZED SUCH THAT
C         KX=1.0 AND KY=1.0 CORRESPOND TO THE VISIBLE
C         REGION OF WAVENUMBER SPACE.
C     FMAX IS AN INPUT-OUTPUT VARIABLE.  IF FMAX IS LESS THAN OR
C         EQUAL TO ZERO ON INPUT, THE FIELD ARRAY IS NORMALIZED
C         FROM ZERO TO ONE AND FMAX IS THE NORMALIZING FACTOR.
C         IF FMAX IS GREATER THAN ZERO ON INPUT IT REMAINS
C         UNCHANGED AND IS USED AS THE NORMALIZING FACTOR.
C     IPWR DETERMINES WHICH POWER COMPONENT WILL BE USED IN THE
C         FAR FIELD CALCULATIONS.  IPWR=1 FOR ELEVATION COMPONENTS,
C         IPWR=2 FOR AZIMUTH COMPONENTS, AND IPWR=3 FOR TOTAL POWER
C         IPWR=4 FOR RIGHT-HAND CIRCULAR POLARIZATION COMPONENTS
C         IPWR=5 FOR LEFT-HAND CIRCULAR POLARIZATION COMPONENTS
C     RADIUS SPECIFIES THE RADIUS OF THE FAR FIELD SPHERE IN
C         CENTIMETERS
C
      REAL FIELD(NX,NY)
      REAL K,KX,KY,KZ,KXINC,KYINC,KXMAX,KYMAX
      COMPLEX XFIELD(NX,NY),YFIELD(NX,NY),Q,C,EZ,EX,EY
      COMPLEX HTHETA,HPHI,HX,HY,HZ
      IM=1+NX/2
      JM=1+NY/2
      SR2=SQRT(2.)
      HTHETA=CMPLX(1.,0.)/SR2
      HPHI=CMPLX(0.,1.)/SR2
      IF (IPWR.EQ.5) HPHI=-HPHI
      IF(IPWR.GE.1.AND.IPWR.LE.5) GO TO 101
```

```
      WRITE(6,100)
  100 FORMAT(1H1,3X,"VALUE ASSIGNED TO THE ARGUMENT IPWR IN SUBROUTINE
     -FAR IS NOT ALLOWED.  IPWR=3 ASSUMED.")
      IPWR=3
  101 CONTINUE
      PI=3.1415926535898
      K=2*PI*FGHZ/29.97925
      D=(0.0,1.0)
      NX2=NX/2
      NY2=NY/2
      KXINC=0.0
      KYINC=0.0
      IF(NX2.EQ.0) GO TO 1
      KXINC=KXMAX/NX2
    1 IF(NY2.EQ.0) GO TO 2
      KYINC=KYMAX/NY2
    2 CONTINUE
C
C         CALCULATE THE POWER PATTERN ON A SPHERE.
C
      R=RADIUS
      RE=AMOD(K*R,2.0*PI)
      C=Q*K*CEXP(-D*RE)/R
      DO 6 I=1,NX,1
      DO 6 J=1,NY,1
      KX=(I-NX2-1.0)*KXINC
      KY=(J-NY2-1.0)*KYINC
      KZ=1.0-KX**2-KY**2
      IF(KZ.LE.0.0) GO TO 5
      KZ=SQRT(KZ)
      D=SQRT(1.-KY**2)
      EZ=-C*(KX*XFIELD(I,J) +KY*YFIELD(I,J))
      EX=C*KZ*XFIELD(I,J)
      EY=C*KZ*YFIELD(I,J)
      XFIELD(I,J)=EX
      YFIELD(I,J)=EY
      IF(IPWR.EQ.1) FIELD(I,J)=CABS(-EX*KY*KX/D+EY*D-EZ*KZ*KY/D)**2
      IF(IPWR.EQ.2) FIELD(I,J)=CABS(-EY*KZ/D-EZ*KX/D)**2
```

```
      IF(IPWR.EQ.3) FIELD(I,J)=CABS(EX)**2+CABS(EY)**2+CABS(EZ)**2
      IF(IPWR.GE.1.AND.IPWR.LE.3) GO TO 6
      IF (I.EQ.IM.AND.J.EQ.JM) GO TO 7
      RAD=SQRT(KX**2+KY**2)
      HX=(HTHETA*KX*KZ-HPHI*KY)/RAD
      HY=(HTHETA*KY*KZ+HPHI*KX)/RAD
      HZ=-HTHETA*RAD
      GO TO 10
    7 HX=HTHETA
      HY=HPHI
      HZ=CMPLX(0.,0.)
   10 FIELD(I,J)=CABS(EX*HX+EY*HY+EZ*HZ)**2
      GO TO 6
    5 FIELD(I,J)=0.0
    6 CONTINUE
C
C           NORMALIZE THE POWER PATTERN.
C
      IF(FMAX.GT.0.0)   GO TO 9
      DO 9  I=1,NX
      DO 8  J=1,NY
      R=FIELD(I,J)
      IF(R.GT.FMAX) FMAX=R
    8 CONTINUE
    9 CONTINUE
      DO 11 I=1,NX
      DO 11 J=1,NY
      FIELD(I,J)=FIELD(I,J)/FMAX
   11 CONTINUE
      RETURN
      END
```

Chapter 21

SUBROUTINE AMPHS

21-1. Purpose: To convert a complex number $c = x + jy$ from rectangular to polar form $c = |c|e^{j\phi}$.

21-2. Usage: CALL AMPHS (C, AMP, PHS)

21-3. Arguments

C - Complex input variable containing the rectangular components of the complex number to be converted; i.e., C = CMPLX(X,Y).

AMP - Real output variable equal to $\sqrt{x^2+y^2}$.

PHS - Real output variable equal to the phase angle ϕ in degrees.

21-4. Comment

The intrinsic Fortran function ATAN2 is used to compute PHS.

21-5. Program Flow: See listing below.

21-6. Test Case: None

21-7. References: None

21-8. Program Listing: See following page.

```
      SUBROUTINE AMPHS(C,AMP,PHS)
      COMPLEX C
      DATA PI/3.1415926/
      AMP=CABS(C)
      X=REAL(C)
      Y=AIMAG(C)
      IF (ABS(X).LT.1E-18) GO TO 2
      PHS=ATAN2(Y,X)*180./PI
      RETURN
    2 PHS=90.
      IF (Y.LT.0.) PHS=-90.
      IF (AMP.LT.1E-18) PHS=-180.
      RETURN
      END
```

Chapter 22

SUBROUTINE DBPV

22-1. Purpose: To convert a real array of linear values, normalized to lie between zero and unity, to decibels.

22-2. Usage: CALL DBPV (FIELD, NX, NY, IPV)

22-3. Arguments

FIELD - Real input/output array of NX by NY elements: on input, it contains the values to be converted; on output, it contains the corresponding decibel values on the range (-40, 0). All input values less than 10^{-2} are set to -40 dB on output.

NX, NY - Integer input variables which specify the size of the array FIELD.

IPV - Integer input variable which specifies whether the input values in FIELD represent power (IPV=1) or voltage (IPV=2). If IPV=1, $F(I, J) = 10 \log_{10} F(I, J)$ is returned; if IPV=2, $F(I, J) = 20 \log_{10} F(I, J)$ is returned.

22-4. Comments

It is intended that the input array FIELD be normalized prior to the call to Subroutine DBPV.

22-5. Program Flow: See listing below.

22-6. Test Case: None

22-7. References: None

22-8. Program Listing: See following page.

```
      SUBROUTINE DBPV(FIELD,NX,NY,IPV)
C  MODIFIED BY GKH 4/78 TO PERMIT POWER (IPV=1) OR VOLTAGE (IPV=2) DB.
C
C  SUBROUTINE DB CONVERTS AN INPUT ARRAY (FIELD(NX,NY)) OF
C  VOLTAGE OR POWER VALUES TO DECIBLES AND RETURNS DB VALUES IN THE
C  SAME ARRAY.
C  ALL VALUES OF POWER LESS THAN 40 DB DOWN ARE SET EQUAL TO  -40DB
C
      DIMENSION FIELD(NX,NY)
      DO 10  I=1,NX
      DO 10  J=1,NY
      IF (IPV.EQ.2) FIELD(I,J)=FIELD(I,J)**2
      IF(FIELD(I,J).LE.1E-4) FIELD(I,J)=1E-4
      FIELD(I,J)=10.0*ALOG10(FIELD(I,J))
   10 CONTINUE
      RETURN
      END
```

Chapter 23

SUBROUTINE NORMH

23-1. Purpose: To normalize a two-dimensional real array of field values so that all values in the array lie between zero and unity.

23-2. Usage: CALL NORMH (FIELD, IMAX, JMAX, LDB)

23-3. Arguments

FIELD	-	Real array of IMAX by JMAX elements. On input, it contains the field values expressed as non-negative real linear amplitude or as amplitude in decibels. On output, the linear amplitudes are replaced by their scaled values FIELD(I,J)/FMAX, where FMAX is the maximum amplitude value in the array; the logarithmic amplitude values are replaced by (FIELD(I,J)+40.)/40., where -40 decibels is assumed to be the lower bound on the original data.
IMAX,JMAX	-	The number of elements in FIELD.
LDB	-	A logical variable set TRUE if the values in FIELD are in decibels.

23-4. Comments and Method

A function $f(x,y)$ of two variables having minimum value f_{min} and maximum value f_{max} may be normalized to $0 \leq f_n(x,y) \leq 1$ according to

$$f_n(x,y) = \frac{f(x,y) - f_{min}}{f_{max} - f_{min}} \qquad (1)$$

provided that the denominator is not zero. In this procedure, the $f_n=0$ corresponds to $f=f_{min}$, and $f_n=1$ corresponds to $f=f_{max}$.

When f(x,y) represents a linear (vice logarithmic) variable, it is desirable to force f_{min} to be zero if the minimum value of f is actually greater than zero. In this special case, f_n becomes

$$f_n(x,y) = \frac{f(x,y)}{f_{max}} \qquad (2)$$

Equation (2) is also used to treat the special case of $f_{max} - f_{min} \simeq 0$; however if $|f_{max}|<1$, f_{max} is set equal to ±1., where the sign used is that of f_{max}. This refinement has the effect of producing a constant function whose value lies between zero and unity; without it, f_n would be simply set to unity or division by zero may result.

When f(x,y) represents a logarithmic variable, such as the amplitude in decibels of an electromagnetic field, all of the foregoing discussion applies; however, a minimum value f_{min} must be imposed. If $f_{min}<-40$, f_{min} is set equal to -80 (decibels); otherwise, a -40 decibel level is assumed. A value of f_{max} equal to zero decibel is also assumed.

23-5. Program Flow

Lines 9-16:	Find minimum MN and maximum MX values of data in FIELD; form their difference DR=MX-MN.
Line 17:	If array values are in decibels, go to 50.
Line 18:	If all values in the array are the same, go to 25 and scale the data to lie between zero and unity (Lines 28-37).

Lines 19-27: If all linear amplitude values in FIELD are not identical, scale the data according to FIELD(I,J) = (FIELD(I,J) - Min. Value)/(Maximum Value - Minimum Value).

Line 38: If values in FIELD are in decibels, and the minimum value is less than -41dB, then assume a -80dB lower bound, go to 60 (Lines 47-52), and scale the data according to (FIELD(I,J) + 80.)/80.

Lines 39-46: Scale the data according to a -40dB lower bound; i.e., (FIELD(I,J) + 40.)/40.

Lines 53-54: Write MN and MX.

23-6. Test Case: See Chapter 2.

23-7. References: None.

23-8. Program Listing: See following pages.

```
      SUBROUTINE NORMH(FIELD,IMAX,JMAX,LDB)
C   MODIFIED BY GKH 4/78 TO CAUSE PROPER NORMALIZATION OF BOTH
C   LINEAR AND DB ARRAYS.
C
C          NORMALIZE FIELD SO THAT ALL VALUES ARE BETWEEN ZERO AND ONE.
C
      REAL MN,MX,FIELD(IMAX,JMAX)
      LOGICAL LDB
      MX=FIELD(1,1)
      MN=MX
      DO 20  I=1,IMAX
      DO 20  J=1,JMAX
      MN=AMIN1(MN,FIELD(I,J))
      MX=AMAX1(MX,FIELD(I,J))
   20 CONTINUE
      DR=MX-MN
      IF (LDB) GO TO 50
      IF (DR.LT.1E-18) GO TO 25
      TMN=MN
      IF (MN.GT.0.) TMN=0.
      TDR=DR
      IF (MN.GT.0.) TDR=MX
      DO 21  I=1,IMAX
      DO 21  J=1,JMAX
      FIELD(I,J)=(FIELD(I,J)-TMN)/TDR
   21 CONTINUE
      GO TO 35
C   CASE WHERE ALL VALUES ARE THE SAME:
   25 TMX=MX
      IF (ABS(MX).LT.1.0) TMX=SIGN(1.,MX)
      DO 30 I=1,IMAX
      DO 30 J=1,JMAX
      FIELD(I,J)=FIELD(I,J)/TMX
C   FIELD IS FILLED WITH SAME VALUES SCALED BETWEEN ZERO AND UNITY.
      IF (FIELD(I,J).LT.0.) FIELD(I,J)=0.
   30 CONTINUE
      GO TO 35
   50 IF (MN.LT.-41.) GO TO 60
```

```
C  ASSUME 0 TO -40. SCALE:
      DO 55 I=1,IMAX
      DO 55 J=1,JMAX
      FIELD(I,J)=(FIELD(I,J)+40.)/40.
      IF (FIELD(I,J).LT.0.) FIELD(I,J)=0.
      IF (FIELD(I,J).GT.1.) FIELD(I,J)=1.
   55 CONTINUE
      GO TO 35
   60 DO 65 I=1,IMAX
      DO 65 J=1,JMAX
      FIELD(I,J)=(FIELD(I,J)+80.)/80.
      IF (FIELD(I,J).LT.0.) FIELD(I,J)=0.
      IF (FIELD(I,J).GT.1.) FIELD(I,J)=1.
   65 CONTINUE
   35 WRITE(6,40) MN,MX
   40 FORMAT(//"  SUBROUTINE NORM:  MIN= ",E10.3,"  MAX= ",E10.3//)
      RETURN
      END
```

Chapter 24

SUBROUTINE CNPLTH AND FUNCTION PSI

24-1. Purpose: To plot (Calcomp) single dimensional far field patterns at constant wavenumber k_{fix}.

24-2. Usage: CALL CNPLTH (FIELD, N, KMAX, KCNTR, KFIX)

$$PSI = ATAN2 \quad (K/\sqrt{1. - K^2 - K_{fix}^2})$$

24-3. Arguments

FIELD - Real input array of N elements containing the field values in decibels but normalized so that -40 dB corresponds to 0 and 0 dB corresponds to unity on the normalized scale.

N - Integer input variable which specifies the number of elements in FIELD.

KMAX - Real input variable equal to the half width of the wavenumber range corresponding to the array elements 1 through N of the array FIELD; i.e., the increment in wavenumber corresponding to the distance between the Ith and (I+1)st element is 2 KMAX/N.

KCNTR - Real input variable equal to the wavenumber coordinate of the (N/2 + 1)st element of the array FIELD. FIELD(1) has wavenumber coordinate KCNTR - KMAX.

KFIX - Real input variable equal to the fixed value of the other wavenumber coordinate. For example, if k_x varies, then k_y = KFIX.

24-4. Comment and Method

Let $F(k_x, k_y)$ represent the far-field power pattern of an antenna where k_x and k_y are normalized wavenumbers as defined in Chapter 20. A pattern cut at constant wavenumber is a conical cut about the real axis; e.g., a k_x = constant cut is a conical cut about the x axis of the coordinate system.

For principal plane cuts, $k_x = 0$ yields an E-plane pattern as defined in Figure 2-3; $k_y = 0$ yields an H-plane pattern. For principal plane cuts, KCNTR = 0 and KFIX = 0.

The plotting commands are set up to produce a 4" X 8" rectangular pattern plot on a standard pattern scale. The plot is positioned on the paper to give margins of 2" on the left, 1" on the right, and 2.25" from the bottom and, hence, is suitable for direct use as a figure in a technical report.

24-5. Program Flow: See listing below.

24-6. Test Case: See Chapter 2 and pattern plots in Appendices B and D.

24-7. References: None

24-8. Program Listing: See following pages.

```
      SUBROUTINE CNPLTH(FIELD,N,KMAX,KCNTR,KFIX)
C  MODIFIED BY GKH 4/28/78 TO GIVE 4 X 8 SA PLOTS WITH MARGINS
C  OF 2" FROM LEFT, 2.25 FROM BTM, AND 1" ON RIGHT.
C
C    THIS SUBROUTINE PLOTS SINGLE DIMENSIONAL FAR FILED PATTERNS
C     THE PLOTS ARE CONSTANT WAVENUMBER PLOTS WHICH CORRESPOND
C     TO CONICAL FAR FIELD PATTERNS
C     FIELD(N) IS A ONE DIMENSIONAL FAR FIELD POWER PATTERN
C     NORMALIZED FROM ZERO(CORRESPONDING TO -40 DB) TO ONE
C     (CORRESPONDING TO 0 DB)
C     KMAX IS THE HALF WIDTH OF THE WAVENUMBER REGION OF FIELD
C     KCNTR IS THE CENTER WAVENUMBER COORDINATE OF THE INPUT FIELD
C     FIELD(1) HAS A WAVENUMBER COORDINATE KCNTR-KMAX
C     FIELD(N/2+1) HAS WAVENUMBER COORDINATE KCNTR
C     KFIX IS THE FIXED VALUE OF THE OTHER WAVENUMBER COORDINATE
C
      REAL FIELD(N),K,KMAX,KCNTR,KFIX
      PSIMIN=PSI(KCNTR-KMAX,KFIX)
      PSIMAX=PSI(KCNTR+KMAX-2*KMAX/N,KFIX)
      PSIMID=PSI(KCNTR,KFIX)
      DELPSI=2*AMAX1(PSIMID-PSIMIN,PSIMAX-PSIMID)
      ISCALE=360
      IF(DELPSI.LE.60) ISCALE=60
      IF(DELPSI.LE.10) ISCALE=10
C  INITIALIZE FACTOR TO UNITY AND DRAW LEFT MARGIN FOR GUIDE LATER.
      CALL FACTOR(1.)
      CALL PLOT(0.,0.0,-3)
      CALL PLOT(0.,8.5,2)
      CALL PLOT(0.,0.,3)
C  SET LOGICAL ORIGIN OF SA PLOT:
      CALL PLOT(2.,2.25,-3)
C  PLOT AT .4 SCALE FACTOR OF FULL SIZE SA PLOT (10 X 20):
      CALL FACTOR(.4)
C
C     DRAW RECTANGULAR PERIMETER BOX
      CALL PLOT(0.0,0.75,3)
      CALL PLOT(0.0,10.625,2)
      CALL PLOT(20.0,10.625,2)
```

```
      CALL PLOT(20.0,.75,2)
      CALL PLOT(0.0,.75,2)
C
C     PLOT HORIZONTAL DECIBEL SCALES
      Y=0.75+2.46875
      DO 1 I=2,4,1
      DB=40.000001-(I-1)*10.0
      CALL PLOT(0.0,Y,3)
      CALL PLOT(5.0,Y,2)
      CALL NUMBER(5.0633,Y-0.07,0.14,DB,0.0,-1)
      CALL PLOT(5.3333,Y,3)
      CALL PLOT(15.3333,Y,2)
      CALL NUMBER(15.3833,Y-0.07,0.14,DB,0.0,-1)
      CALL PLOT(15.6666,Y,3)
      CALL PLOT(20.0,Y,2)
    1 Y=Y+2.46875
C
C     PLOT HORIZONTAL ANGLE TICK MARKS
      X=0.6666667
      DO 2 I=1,29,1
      CALL PLOT(X,0.75,3)
      CALL PLOT(X,0.85,2)
    2 X=X+0.66666667
C
C     PLOT HORIZONTAL ANGLE SCALE
      DO 3 I=1,9,1
      ANG=IABS(5-I)*ISCALE/10.0+0.0001
      DX=0.18
      IF(ANG.LT.100.0)DX=0.11
      IF(ANG.LT.10.0)DX=0.04
      X=I*2.0-DX
      Y=0.90
      CALL NUMBER(X,Y,0.14,ANG,0.0,-1)
    3 CONTINUE
C
C     DIGRESS TO PLOT INTERIOR VERTICLE DECIBLE SCALES AND LABLES
      DX=-1.0
      X= 5.3333
```

```
      DO 6 I=1,2,1
      Y=0.75
      CALL PLOT(X,Y,3)
      CALL PLOT(X,10.625,2)
      CALL SYMBOL(X-DX*0.14+0.07,3.85,0.14, 27HRELATIVE POWER ONE WAY (D
     $B),90.0,27)
      DO 5 J=1,4,1
      DO 4 L=1,4,1
      DB=10-2*L+0.0001
      Y=Y+9.875/20.0
      CALL PLOT(X,Y,3)
      CALL PLOT(X+DX*0.07,Y,2)
      CALL NUMBER(X+DX*0.16666-0.04,Y-0.07,0.14,DB,0.0,-1)
    4 CONTINUE
    5 Y=Y+9.875/20.0
      X=14.6667
    6 DX=-DX
C
C    PLOT CONE ANGLE AND CENTER OF ROTATION ANGLE
      CONE=ACOS(KFIX)*180/3.14159265358 98
      CALL SYMBOL(6.5,0.54,0.14,13HCONE ANGLE = ,0.0,13)
      CALL NUMBER(999.,999.,0.14,CONE,0.0,1)
      CALL SYMBOL(999.,999.,0.14,29H      CENTER ROTATION ANGLE = ,0.0,
     $29)
      CALL NUMBER (999.,999.,0.14,PSIMID,0.0,1)
C
C     PLOT PATTERN
      IPEN=3
      DO 10 I=1,N,1
      K=KCNTR+(I-N/2.-1)*KMAX*2/N
      A=PSI(K,KFIX)-PSIMID
      X=10.0+20.0*A/ISCALE
      Y=FIELD(I)
      IF(Y.LT.0.0) Y=0.0
      IF(Y.GT.1.0) Y=1.0
      Y=Y*9.875+0.75
      IF(X.LT.0.0)X=0.0
      IF(X.GT.20.0) X=20.0
```

```
      CALL PLOT(X,Y,IPEN)
      IPEN=2
   10 CONTINUE
C  RESTORE FACTOR AND CONCLUDE PLOT AT BTM RT CORNER OF PAGE.
      CALL FACTOR(1.0)
      CALL PLOT(9.,-2.25,-3)
      RETURN
      END
```

```
FUNCTION PSI(K,KFIX)
REAL K,KFIX,KZ
KZ=1.-K**2-KFIX**2
IF (KZ.LE.0.) KZ=0.
KZ=SQRT(KZ)
PSI=ATAN2(K,KZ)*180./3.14159265 3
RETURN
END
```

Chapter 25

SUBROUTINES PLT3DH AND PLTT

25-1. Purpose: To plot (Calcomp) the two-dimensional array FIELD (I, J).

25-2. Usage: CALL PLT3DH (XSIZE, YSIZE, HEIGHT, FIELD, IMAX, JMAX, NMZ, LDB)

25-3. Arguments

XSIZE, YSIZE, HEIGHT	-	Real input variables in inches defined on Figure 1.
FIELD, IMAX, JMAX	-	Real input array of IMAX by JMAX elements containing the values to be plotted. These values must be normalized to the range (0, 1) before plotting.
NMZ	-	Logical input variable. If NMZ = .TRUE., the array FIELD will be normalized with respect to its own maximum value; if NMZ = .FALSE., no normalization will be done.
LDB	-	Logical input variable required by Subroutine NORMH (Chapter 23).

25-4. Comments

In Figure 25-1, the axes and labels shown are not produced by the subroutine; these axes are presented to demonstrate the perspective of the plot and to identify its dimensions. Report size plots will be produced suitable for one 8 1/2" X 11" page when FACTOR = 1.0 and

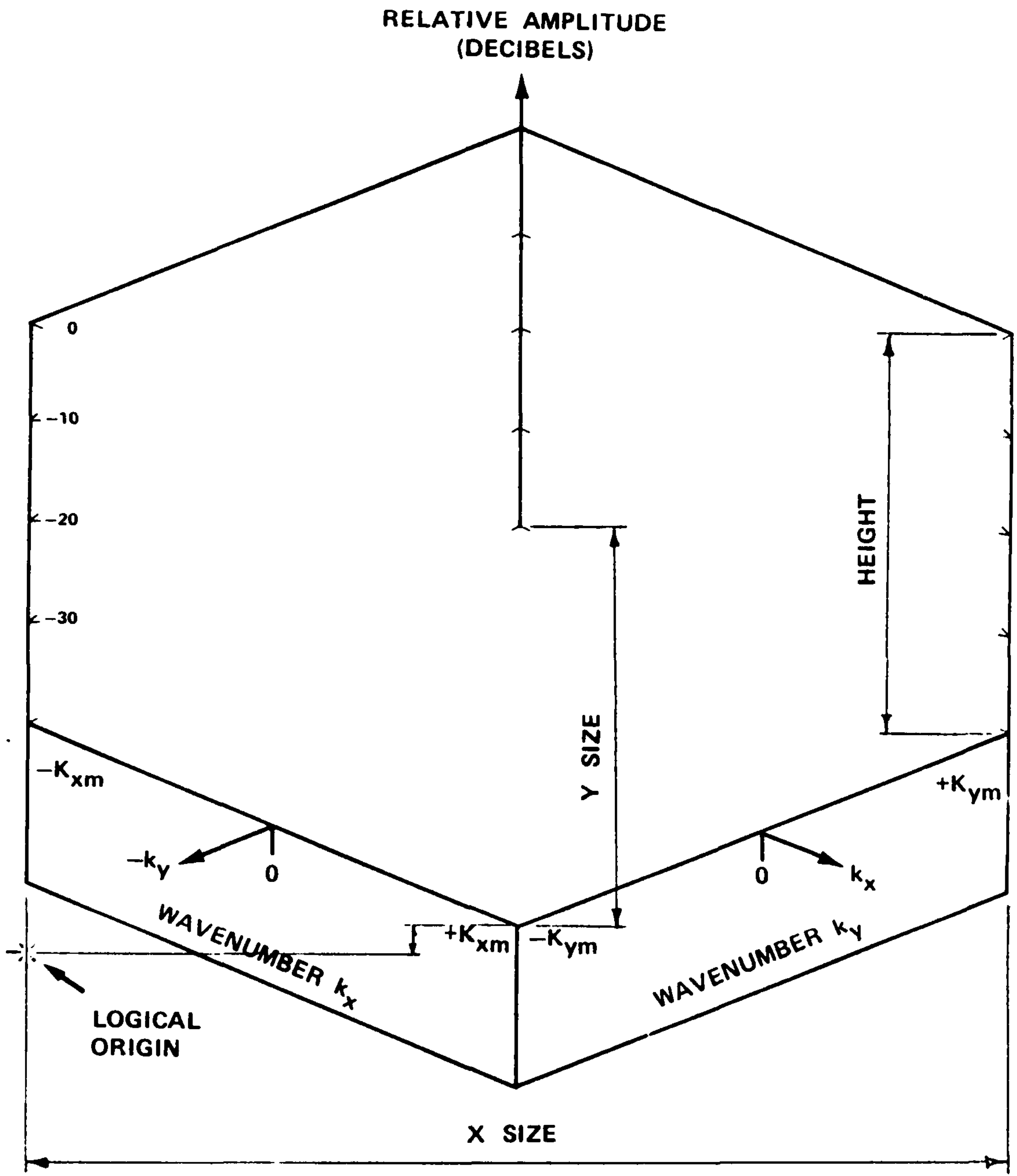

Figure 25–1. Dimensions of Three–Dimensional Plot.

$$XSIZE \approx 6.0"$$

$$YSIZE \approx 2.5"$$

$$HEIGHT \approx 2.5"$$

Margins in this case will be 1.5" on the left, 1" on the right, and 4.25" from the bottom of the plot paper. Margin lines are provided on the plot paper to outline the 8 1/2" X 11" page. Also, the plot itself can be carefully cut from the plot paper and cemented onto a set of axes as has evidently been done in Appendices B and D.

25-5. Program Flow: See listing below.

25-6. Test Case: See Chapter 2 and Appendices B and D.

25-7. References: None.

25-8. Program Listing: See following pages.

```
      SUBROUTINE PLT3CH(XSIZE,YSIZE,HEIGHT,FIELD,IMAX,JMAX,NMZ,LDB)
C  LDB IS REQUIRED BY SUBR NORMH TO SPECIFY IF ARRAY FIELD
C  IS IN DB (TRUE) OR NOT (LDB=.FALSE.).
C  MODIFIED BY GKH 4/28/78 TO GIVE REPORT SIZE PLOTS WHEN
C  XSIZE=6.0, YSIZE=2.5, HEIGHT=2.5.  I.E., LEFT MARGIN OF
C  1.5", RIGHT OF 1" AND 4.25" FROM BTM MARGIN.
C
C          XSIZE IS THE MAXIMUM LENGTH OF THE PLOT IN INCHES.
C          YSIZE IS THE MAXIMUM WIDTH OF A ZERO PLOT IN INCHES.
C          THE SUM OF 1/2 INCH + YSIZE + HEIGHT MUST BE LESS THAN
C              OR EQUAL TO THE PAPER WIDTH.
C          FIELD(IMAX,JMAX) IS THE TWO-DIMENSIONAL REAL ARRAY TO
C              BE PLOTTED. IF FIELD IS NOT NORMALIZED ON INPUT, NMZ
C              MUST BE .TRUE.   .
C          NMZ(NORMALIZE) IS A LOGICAL INPUT VARIABLE.  IF ITS VALUE
C              IS .TRUE. THE VALUES IN FIELD WILL BE REPLACED WITH
C              THEIR NORMALIZED(ZERO TO ONE) COMPONENTS.
C
      REAL FIELD(IMAX,JMAX),HID(128)
      LOGICAL NMZ,LDB
      REAL LASTX,LASTY,LASTH,LASTHM
      IF(NMZ)  CALL NORMH(FIELD,IMAX,JMAX,LDB)
      XPAGE=0.0
      YPAGE=0.0
      LASTHM=0.0
      NIJ=IMAX+JMAX
      RI=IMAX-1.0
      RJ=JMAX-1.0
C  INITIALIZE FACTOR TO UNITY, DRAW LEFT MARGIN,AND SET LOGICAL ORIGIN*
      CALL FACTOR(1.0)
      CALL PLOT(0.,0.0,-3)
      CALL PLOT(0.,11.,2)
      CALL PLOT(0.,0.,3)
      CALL PLOT(1.5,3.75,-3)
      DO 1 I=1,NIJ
    1 HID(I)=-0.5
      DO 7 J=1,JMAX
      AJ=J-1.0
```

```
      DO 7 I=1,IMAX
      AI=I-1.0
      LASTX=XPAGE
      YPAGE=(AJ+AI)*YSIZE/(RI+RJ)
      LASTY=YPAGE
      YPAGE=(AJ*RI/RJ-AI*RJ/RI+RJ)*YSIZE/(RJ+RI)+HEIGHT*FIELD(I,J)
      LASTH=LASTHM
      LASTHM=HID(I+J)
      IF(YPAGE-HID(I+J)) 5,5,2
    2 IF(I.NE.1) GO TO 3
      CALL PLTT(XPAGE,YPAGE,3)
      IPEN=2
      GO TO 4
    3 CALL PLTT(XPAGE,YPAGE,IPEN)
      IPEN=2
    4 HID(I+J)=YPAGE
      GO TO 7
    5 IF(I.EQ.1) IPEN=3
      IF(IPEN.EQ.3) GO TO 6
      X1N=LASTX*HID(I+J)-LASTH*XPAGE-LASTX*YPAGE+XPAGE*LASTY
      X1D=HID(I+J)-LASTH-YPAGE+LASTY
      X1=X1N/X1D
      Y1=(X1*(HID(I+J)-LASTH)+LASTH*XPAGE-LASTX*HID(I+J))/(XPAGE-LASTX)
      CALL PLTT(X1,Y1,2)
      IPEN=3
    6 CALL PLTT(XPAGE,YPAGE,IPEN)
    7 CONTINUE
      DO 8 I=1,NIJ
    8 HID(I)=-0.5
      DO 16 II=1,IMAX,1
      I=IMAX-II+1
      AI=I-1
      DO 16 J=1,JMAX
      AJ=J-1
      LASTX=XPAGE
      XPAGE=(AJ+AI)*XSIZE/(RI+RJ)
      LASTY=YPAGE
      YPAGE=(AJ*RI/RJ-AI*RJ/RI+RJ)*YSIZE/(RJ+RI)+HEIGHT*FIELD(I,J)
```

```
      LASTH=LASTHM
      LASTHM=HID(I+J)
      IF(YPAGE-HID(I+J)) 13,14,9
    9 IF(J.NE.1) GO TO 10
      CALL PLTT(XPAGE,YPAGE,3)
      IPEN=2
      GO TO 12
   10 IF(IPEN.EQ.2) GO TO 11
      X1N=LASTX*YPAGE-LASTY*XPAGE-LASTX*HID(I+J)+XPAGE*HID(I+J-1)
      X1D=YPAGE-LASTY-HID(I+J)+HID(I+J-1)
      X1=X1N/X1D
      Y1=(X1*(YPAGE-LASTY)+LASTY*XPAGE-LASTX*YPAGE)/(XPAGE-LASTX)
      CALL PLTT(X1,Y1,3)
      IPEN=2
   11 CALL PLTT(XPAGE,YPAGE,IPEN)
   12 HID(I+J)=YPAGE
      GO TO 16
   13 IPEN=3
      GO TO 15
   14 IF(J.EQ.1) IPEN=3
   15 CALL PLTT(XPAGE,YPAGE,IPEN)
   16 CONTINUE
C     CONCLUDE PLOT AT RT BTM CORNER OF REPORT PAGE.
      CALL PLOT(XSIZE+1.,-3.75,-3)
      RETURN
      END
```

```
      SUBROUTINE FLTT(X,Y,IPEN)
C
C          SUBROUTINE PLTT ELIMINATES MOVING PEN FOR HIDDEN LINES.
C
      XLAST=XN
      YLAST=YN
      ILAST=IN
      XN=X
      YN=Y
      IN=IPEN
      IF(IPEN.EQ.2.AND.ILAST.EQ.2) CALL PLOT(X,Y,IPEN)
      IF(IPEN.EQ.2.AND.ILAST.EQ.3) CALL PLOT(XLAST,YLAST,ILAST)
      IF(IPEN.EQ.2.AND.ILAST.EQ.3) CALL PLOT(X,Y,IPEN)
      IF(IPEN.NE.2.AND.IPEN.NE.3) CALL FLCT(X,Y,IPEN)
      RETURN
      END
```

Chapter 26

SUBROUTINE FFTA

26-1. Purpose: To compute the Discrete Fourier Transform (DFT) or its inverse of a sequence of complex numbers consisting of 2^N elements, where N is an integer. The Cooley-Tukey algorithm is used to perform computations in place to speed up the computations and to return the transformed values in the input array.

26-2. Usage: CALL FFTA (FIELD, NEXP, IBMISN)

26-3. Arguments

FIELD - Complex array of 2** NEXP elements: on input it contains the sample data to be transformed; on output it contains the transformed data. See below for ordering of data.

NEXP - Integer exponent; e.g., for 64 elements in FIELD, NEXP=6.

IBMISN - Integer parameter which controls operation:
IBMISN = 3 performs the inverse DFT
IBMISN $\neq$ 3 performs the DFT as defined in 4 below.

26-4. Comments and Method

a. Subroutine FFTA is machine-dependent in that the bit reversed number, IFLIP, must be generated using Fortran instructions which are peculiar to a particular machine. Also, the word length must be taken into account. Lines 38-42 of the attached program listing are used to effect the desired operation for the CDC Cyber 70 (60-bit word, numbered

0 through 59 from right to left with Bit 0 being the least significant):

```
      IFLIP=0
      DO 4 II=1, IEXP, 1
      J=60 - II
      IFLIP=2*IFLIP + AND (SHIFT(I,1+J), 1B)
    4 CONTINUE
```

The SHIFT(I,1+J) operation shifts the bits of the integer I to the left by 1+J bit positions. The AND operation strips off the right most bit of the shifted result. E.G., when II=1, the right most bit of I (Bit 0) is extracted from I by the AND(SHIFT) operation. The current value of IFLIP is then shifted one bit to the left by the 2*IFLIP operation. The two results are then added together. A total of NEXP bits are extracted, starting with Bit 0, followed by Bits 1, 2,...(NEXP-1).

The net result of these operations is to take the NEXP-bit binary representation of the array element number I, reverse the order of the bits, and right justify the result. Array elements in FIELD numbered I and IFLIP are then interchanged if I>IFLIP. The first and last elements of FIELD always remain in place. The array elements are rearranged in this manner so that they will be ordered after transforming [1].

b. To explain the ordering of the data in the complex array FIELD, it is convenient to consider the specific example of using FFTA to compute the Fourier transform G(f) of a time function g(t) as defined by

$$G(f) = \int_{-T_{max}}^{T_{max}} g(t)\, e^{-j2\pi ft}\, dt \qquad (1)$$

and as approximated by

$$G(f) \approx \Sigma\, g(t_i)\, e^{-j2\ \ ft_i}\, \Delta t \qquad (2)$$

where t_i are the equally spaced points along the t axis when g is sampled over the interval $-T_{max} \le t \le T_{max}$.

There are $N=2^{NEXP}$ samples in the input array FIELD(I) corresponding to I=1,N. The first sample (I=1) corresponds to $g(-T_{max})$. The last sample (I=N) corresponds to $g(T_{max}-\Delta t)$. The I=(N/2+1)th sample corresponds to g(0); i.e., the value of g at t=0. The DFT assumes periodicity of the sampled data so that the value at $t=T_{max}$ is identical to that at $t=-T_{max}$. The sample spacing is

$$\Delta t = 2\ T_{max}/N \qquad (3)$$

and corresponds to a folding frequency f_{max} of

$$f_{max} = 1/2\Delta t \qquad (4)$$

On output, the array FIELD contains the frequency components G(f) at N equally spaced frequencies Δf over the band $-f_{max} \le f \le f_{max}$, where I=1 corresponds to $f=-f_{max}$, I=(N/2+1) to f=0, and I=N to $f=f_{max}-\Delta f$, where

$$\Delta f = 2\ f_{max}/N \qquad (5)$$

and where

$$T_{max} = \frac{1}{2\Delta f} \tag{6}$$

Also, by the inversion integral [2],

$$g(t) = \int_{-f_{max}}^{f_{max}} G(f)\ e^{+j2\pi ft}\ df \approx \Delta f \sum_p G(f_p)\ e^{j2\pi f_p t} \tag{7}$$

This version of Subroutine FFTA is written so that division by N is done when the Fourier transform (kernel = $e^{-j2\pi ft}$) is computed. When the expression in Equation (3) for Δt is used in (2), there results

$$G(f_p) = 2\ T_{max} \frac{1}{N} \sum_i g(t_i)\ e^{-j2\pi f_p t_i} \tag{8}$$

Transposing 2 T_{max} and using Equation (6) yields

$$\Delta f\ G(f_p) = \frac{1}{N} \sum_i g(t_i)\ e^{-j2\pi f_p t_i} \tag{9}$$

where the righthand side is the definition of the Discrete Fourier Transform as computed by FFTA. Inversely,

$$g(t_i) = \sum_p \Delta f\ G(f_p)\ e^{+j2\pi f_p t_i} \tag{10}$$

which is the Inverse DFT as computed by FFTA.

Conversely, if the original data in the input array FIELD are samples of a frequency spectrum G(f), a similar analysis shows that FFTA computes $\Delta t\ g(t_i)$ as the inverse transform (IBMISN=3); i.e., the time function is modified in amplitude by Δt. Of course, when the forward transform (IBMISN$\neq$3) is performed on this result, the original sampled data $G(f_i)$ are obtained in FIELD on output.

From the above considerations, the following conclusions can be drawn concerning the use of FFTA to compute the Fourier transform G(f) of a windowed time function g(t):

$$G(f_p) = 2\ T_{max} \cdot \mathrm{FFTA}\{g(t_i)\} \tag{11}$$

$$g(t_i) = \frac{1}{2\ T_{max}} \cdot \mathrm{IFFTA}\{G(f_p)\} = \mathrm{IFFTA}\{\Delta f\ G(f_p)\} \tag{12}$$

As an example, let g(t) be the rectangular pulse function which has constant amplitude V_o for $|t| \le t_o$ and which is windowed in the larger time interval $|t| \le T_{max}$. The Fourier transform G(f) is given by [3]

$$G(f) = 2\ t_o V_o \frac{\sin 2\pi f t_o}{2\pi f t_o} \tag{13}$$

Let g(t) be sampled at $N=2^{NEXP}$ points over the interval $|t| \le T_{max}$, and let these sampled points be placed in the array FIELD. Then the spectrum G(f) will be closely approximated at discrete frequencies f_p by

$$G(f_p) \approx 2\ T_{max} * \text{FIELD(I)}$$

where

$$f_p = -f_{max} + (I-1) * \Delta f$$

and where FIELD is the output of FFTA according to CALL FFTA (FIELD, N, 0).

Proper consideration should be given to the sampling of the time function so that the DFT produces a good estimate of the actual integral transform. For example, if $t_o = T_{max}$, and all samples are constant, then the DFT will produce a single nonzero frequency component at f=0 (corresponding to the (N/2+1)th element of FIELD); i.e., a delta function. Such a result follows from the facts that the Fourier transform of a constant $g(t)=V_o$ is $G(f)=V_o\delta(f)$ and that the DFT assumes a periodicity of the sequence of samples provided to it.

Consider the other extreme. Let the pulse g(t) be represented by only one sample at t=0 in the window $|t| \leq T_{max}$. The Fourier transform of $g(t)=V_o\delta(t)$ is $G(f)=V_o$, a constant.

It is clear from the above considerations that the time function must be properly windowed and properly sampled to produce a good estimate of its transform via the DFT. Simply stated, the time function should be sampled at a rate Δt which is twice the highest frequency contained in the function as interpreted by the DFT.

26-5. Program Flow

Lines 22-24: Compute $N=2^{NEXP}$ and set the sign ISN of the exponent in the Fourier kernel.

Lines 26-29: Compute IEXP=NEXP from N. This is a redundant computation made when the original FFT subroutine was modified to conform to the call to a library version on another computer system.

Lines 30-35: Rearrange the order of the input data so that samples for $t \geq 0$ are placed in the lower half of the array, and those for $t < 0$ are placed in the upper half. For a frequency function, the data are rearranged so that the first N/2 points give the components for non-negative frequencies (I=1 corresponds to f=0), and the last N/2 points contain the data for the negative frequencies.

Lines 36-49: Rearrange the data in FIELD so that it will be ordered after transforming as described for Lines 30-35 above.

Lines 50-73: Perform the summation using the Cooley-Tukey algorithm [1].

Lines 74-79: If forward transform is being done, divide all values in FIELD by N.

Lines 80-85: Rearrange the output data in FIELD so that it conforms to that used on input; i.e., $f_i = f_{max} + (I-1)\Delta f$ or $t_i = -T_{max} + (I-1)\Delta t$ as appropriate.

26-6. Test Case

A rectangular pulse function with amplitude $V_o = 100$ was chosen for g(t) with $t_o = .10$ second $T_{max} = 1.60$ seconds, and $N=2048=2^{11}$. The resulting

sample increment Δt and folding frequency f_{max} were 0.116 second 320.0 Hertz, respectively. The comparison of the central nine points of the computed and true frequency spectra were as follows (CDC Cyber 70):

		True G(f)		Computed G(f)	
I	f (Hz)	Amp.	Phase(°)	Amp.	Phase(°)
1021	-1.250	18.006	0.00	18.006	0.35
1022	-0.938	18.863	0.00	18.863	0.26
1023	-0.625	19.490	0.00	19.490	0.18
1024	-0.313	19.872	0.00	19.872	0.09
1025	0.000	20.000	0.00	20.000	0.00
1026	0.313	19.872	0.00	19.872	-0.09
1027	0.625	19.490	0.00	19.490	-0.26
1028	0.938	18.863	0.00	18.863	-.035

26-7. References

1. Cochran, W. T., et al, "What is the Fast Fourier Transform?", Proc, IEEE, 55, pp. 1664-1674.
2. Papoulis, A., The Fourier Integral and Its Application, McGraw-Hill, New York, Ch. 2, 1962.
3. Stein and Jones, Modern Communication Principles, McGraw-Hill, New York, pp. 10-11, 1967.

26-8. Program Listing: See following pages. The second listing, Subroutine FFT, is for use on the IBM 3033 at JHU/APL. It employs the subroutine FFTA available on that system library. Use of this subroutine requires the calls in Subroutines JOYFFT and MAGFFT to be changed from CALL FFTA to CALL FFT.

```
      SUBROUTINE FFTA(FIELD,NEXP,IBMISN)
C   MODIFIED TO SIMULATE FFTA ON IBM 3033 AT APL/JHU
C   DIVISION BY N IS DONE WHEN ISN=-1 (GKH 10 20 76)
C   CDC CONVERSION DONE 1 JUNE 1976
C
C    ****************************************************************
C    *                                                              *
C    *   THIS SUBROUTINE CALCULATES THE FAST FOURIER TRANSFORM OR   *
C    *   THE INVERSE FAST FOURIER TRANSFORM OF AN INPUT COMPLEX     *
C    *   ARRAY FIELD AND RETURNS THE RESULT IN THE SAME ARRAY       *
C    *                                                              *
C    *   N MUST BE AN INTEGER POWER OF TWO                          *
C    *                                                              *
C    *   ISN IS AN INTEGER WHICH MAY BE EITHER ONE OR MINUS ONE     *
C    *   IF ISN IS -1 THE FAST FOURIER TRANSFORM IS CALCULATED      *
C    *   IF ISN IS +1 THE INVERSE FOURIER TRANSFORM IS CALCULATED   *
C    *                                                              *
C    ****************************************************************
C
      COMPLEX FIELD(512)
      COMPLEX  T,F
      N=2**NEXP
      ISN=-1
      IF (IBMISN.EQ.3) ISN=+1
      PI2=6.2831853071796
      IEXP=0
    1 IEXP=IEXP + 1
      M=2**IEXP
      IF(N-M) 10,2,1
    2 N2=N/2
      DO 3 I=1,N2,1
      K=I+N2
      T=FIELD(I)
      FIELD(I)=FIELD(K)
    3 FIELD(K)=T
      N1=N-2
      DO 5 I=1,N1,1
      IFLIP=0
```

```
      DO 4 II=1,IEXP,1
      J=60-II
      IFLIP=2*IFLIP+AND(SHIFT(I,1+J),1B)
    4 CONTINUE
      IF(I.LE.IFLIP) GO TO 5
      I1=I+1
      I2=IFLIP+1
      T=FIELD(I2)
      FIELD(I2)=FIELD(I1)
      FIELD(I1)=T
    5 CONTINUE
      DO 6 I=1,IEXP,1
      NEL=2**I
      NEL2=NEL/2
      NSET=N/NEL
      SI=SIN(PI2/NEL)
      CI=COS(PI2/NEL)
      DO 86 J=1,NSET,1
      INCR=(J-1)*NEL
      SO=0.0
      CO=1.0
      DO 96 II=1,NEL2,1
      J1=II+INCR
      J2=J1+NEL2
      T=FIELD(J1)
      F=FIELD(J2)*CMPLX(CO,ISN*SO)
      FIELD(J1)=T+F
      FIELD(J2)=T-F
      SN=SO*CI+CO*SI
      CS=CO*CI-SO*SI
      CO=CS
      SO=SN
   96 CONTINUE
   86 CONTINUE
    6 CONTINUE
      IF(ISN.GT.0) GO TO 8
C   DIVISION BY N IS DONE FOR FORWARD TRANSFORM (GKH 10-20-76)
      DO 7 I=1,N,1
```

```
      FIELD(I)=FIELD(I)/N
    7 CONTINUE
    8 CONTINUE
      DO 9 I=1,N2,1
      K=I+N2
      T=FIELD(I)
      FIELD(I)=FIELD(K)
      FIELD(K)=T
    9 CONTINUE
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FFT(FIELD,NEXP,IBMISN)
C  MODIFIED TO UTILIZE FFTA ON IBM 3033 AT APL/JHU BY SKH FEB 80.
C   ***************************************************************
C   *                                                             *
C   *   THIS SUBROUTINE CALCULATES THE FAST FOURIER TRANSFORM OR  *
C   *   THE INVERSE FAST FOURIER TRANSFORM OF AN INPUT COMPLEX    *
C   *   ARRAY FIELD AND RETURNS THE RESULT IN THE SAME ARRAY      *
C  *  IBMISN (INTEGER) CONTROLS THE DIRECTION OF THE TRANSFORM:
C  *        =1 FOR FORWARD (NEGATIVE EXPONENTIAL) TRANSFORM
C  *        =3 FOR INVERSE TRANSFORM
C  *  SEE JHU/APL SCIENTIFIC SUBR LIBRARY ROUTINE NO. 6.04.051 FOR
C  *  OTHER VALUES OF IBMISN IN SUBR FFTA USED HEREIN.
C  *  ORDERING OF DATA:
C  *    ON INPUT AND OUTPUT, I=1 CORRESPONDS TO MOST NEGATIVE
C  *    ABSCISSA, I=N/2+1 TO ORIGIN, I=N TO MOST POSITIVE ABSCISSA.
C   ***************************************************************
C
      COMPLEX FIELD(1)
      COMPLEX  T,F
      N=2**NEXP
      DATA PI2/6.2831853071796/
    2 N2=N/2
C  ORDER THE DATA FOR FFTA  I.E., NONNEGATIVE ABSCISSAE CORRESPOND
C  TO I=1 TO N/2, NEGATIVE ABSCISSAE TO I=N/2+1 TO N.
      DO 3 I=1,N2,1
      K=I+N2
      T=FIELD(I)
      FIELD(I)=FIELD(K)
    3 FIELD(K)=T
      MFFTA=NEXP+1
      CALL FFTA(FIELD,MFFTA,IBMISN)
C  REORDER THE DATA FOR OUTPUT.
      DO 9 I=1,N2,1
      K=I+N2
      T=FIELD(I)
      FIELD(I)=FIELD(K)
      FIELD(K)=T
    9 CONTINUE
```

```
RETURN
END
```

Chapter 27

SUBROUTINE MAGFFT

27-1. Purpose: To increase the resolution of a complex array of data points using Fourier interpolation and the Fast Fourier Transform. The number of points in each array must be an integer power of two.

27-2. Usage: CALL MAGFFT (A, NA, B, NB)

27-3. Arguments

A,NA - Complex input array of $NA = 2^M < NB$ data points.

B,NB - Complex output array of $NB = 2^N$ data points.

27-4. Comment and Method

a. Subroutines required: FFTA, PWRTWO

b. By Shannon's sampling theorem, a band-limited function is represented by its samples, and it can be reconstructed at any point from them. The computation of the value of the function at a point other than a sample point is called Fourier interpolation. Such interpolation can be used to increase the resolution of a function.

The Fast Fourier Transform (FFT) can be used to facilitate Fourier interpolation. Briefly, the original function $A(k_x)$, known at NA points on the range $(-K_M, + K_M)$, is transformed to yield $E(x) = F\{A(k_x)\}$ at NA sample points. These NA values of $E(x)$ are then placed in the center of an array containing $NB = 2^N > NA = 2^M$ points to form the function $E'(x)$. This function is then inverse transformed to produce $A(k_x)$ at NB points over the same range $(-K_M, + K_M)$. (Actually, the range is $(-K_M, + K_M - \Delta k)$ since the FFT considers the sampled function to be periodic outside the known range so that the (NB + 1)st point would be the same as the first point in the array.)

27-5. Program Flow: See listing below.

27-6. Test Case: See Chapter 2.

27-7. References: See Chapter 26.

27-8. Program Listing: See following page.

```
      SUBROUTINE MAGFFT(A,NA,B,NB)
      COMPLEX A(NA),B(NB)
      IF (NB.LE.NA) GO TO 1
      GO TO 2
    1 WRITE(6,3)
    3 FORMAT(/" NB IS LESS THAN OR EQUAL TO NA IN MAGFFT "/)
      RETURN
    2 CONTINUE
      NAC=NA/2+1
      NBC=NB/2+1
      N=NBC-NAC
      DO 5 I=1,NB
      B(I)=(0.,0.)
    5 CONTINUE
      CALL PWRTWC(NA,INA)
      CALL FFTA(A,INA,3)
      DO 10 I=1,NA
      J=N+I
      B(J)=A(I)
   10 CONTINUE
      CALL PWRTWC(NB,INB)
      CALL FFTA(B,INB,1)
      RETURN
      END
```

Chapter 28

SUBROUTINE JOYFFT

28-1. Purpose: To compute the two-dimensional Fast Fourier Transform of a complex array of NXI by NYI points and to provide magnification of a specified portion of the transformed data.

28-2. Usage: CALL JOYFFT (INPUT, NXI, NYI, MX, MY, NXC, NYC, OUTPUT, NXO, NYO, XYFFT, NXY, ISN)

28-3. Arguments

INPUT, NXI, NYI	-	Complex input array of NXI by NYI points.
MX, MY	-	Integer input variables, equal to an integer power of two, which specify the magnification in the I and J directions, respectively.
NXC, NYC	-	Integer input variables which specify the center coordinate I = NXC, J = NYC of the sector to be magnified.
OUTPUT, NXO, NYO	-	Complex output array of NXO by NYO points containing the transformed points of the magnified sector.
XYFFT, NXY	-	Complex working array of NXY points.
ISN	-	Integer input variable which specifies the direction of the FFT: ISN = 3 for inverse FFT; ISN = 1 for FFT. See Chapter 26.

28-4. Comment

a. Subroutines required: FFTA, PWRTWO.

b. All integer input variables must be integer powers of 2 and must satisfy the following restrictions:

(1) NXO*NYO $\leq$ NXI*NYI

(2) NXO $\leq$ NXI or NYO $\leq$ NYI

(3) MX*NXI $\leq$ NXY and MY*NYI $\leq$ NXY

28-5. Program Flow: See listing below.

28-6. Test Case: See Chapter 2.

28-7. References: None

28-8. Program Listing: See following pages.

```
      SUBROUTINE JOYFFT(INPUT,NXI,NYI,MX,MY,NXC,NYC,OUTPUT,NXO,NYO,
     $XYFFT,NXY,ISN)
      COMPLEX INPUT(NXI,NYI),OUTPUT(NXO,NYO),XYFFT(NXY)
C
C     ******************************************************************
C     *
C     *   SUBROUTINE JOYFFT CALCULATES THE TWO DIMENSIONAL COMPLEX FAST
C     *   FOURIER TRANSFORM FOR ISN=+1 OR THE INVERSE FAST FOURIER
C     *   TRANSFORM FOR ISN=+3 ,OUTPUT(NXC,NYO), OR A TWO DIMENSIONAL
C     *   COMPLEX ARRAY, INPUT(NXI,NYI)
C     *   JOYFFT ALSO PERMITS CALCULATION OF A MAGNIFIED SECTOR OF THE
C     *   FFT AS FOLLOWS.  MX IS THE MAGNIFICATION FACTOR FOR THE X
C     *   DIMENSION AND MY IS THE MAGNIFICATION FACTOR FOR THE Y
C     *   DIMENSION.  THE CENER COORDINATE OF THE MAGNIFIED SECTOR,
C     *   (NXC,NYC) IS SPECIFIED WITH REFERENCE TO THE UNMAGNIFED FFT
C     *
C     *   THE XYFFT(NXY) ARRAY IS A COMPLEX TEMPORARY STORAGE ARRAY USED
C     *   TO PERFORM THE MAGNIFIED X AND Y SINGLE DIMENSION FFTS
C     *
C     *   FOLLOWING ARE RESTRICTIONS ON THE INPUT AND OUTPUT PARAMETERS
C     *   "<" BELOW MEANS LESS THAN OR EQUAL TO
C     *       NXO*NYO<NXI*NYI
C     *       NXO<NXI OR NYO<NYI
C     *       MX*NXI<NXY AND MY*NYI<NXY
C     *     NYI=2**(ANY NON-NEGATIVE INTEGER)
C     *     NXI=2**(ANY NON-NEGATIVE INTEGER)
C     *     NXO=2**(ANY NON-NEGATIVE INTEGER)
C     *     NYO=2**(ANY NON-NEGATIVE INTEGER)
C     *       MX=2**(ANY NON-NEGATIVE INTEGER)
C     *       MY=2**(ANY NON-NEGATIVE INTEGER)
C     *
C     *   MAGNIFICATION IS NOT PERMITTED FOR INPUT DIMENSIONS OF ONE
C     *
C     *   THE OUTPUT SECTOR MUST BE CONTAINED IN THE MAGNIFIED FFT
C     *         NXC/2<MX*MIN(NXC-1,NXI+1-NXC)
C     *         AND
C     *         NYC/2<MY*MIN   C-1,NYI+1-NYC)
C     *
```

```
C   *   THE INPUT AND OUTPUT ARRAY MAY BE EQUIVALENCED USING THE
C   *   FOLLOWING EQUIVALENCE STATEMENT IN THE MAIN PROGRAM
C   *   EQUIVALENCE(INPUT(1,1),OUTPUT(1,1))
C   ****************************************************************
C
C   NXO*NYO<NXI*NYI
      IF(NXI*NYI.GE.NXO*NYO) GO TO 2
C     WRITE(6,1)
C   1 FORMAT(5X,"THE SIZE OF THE OUTPUT ARRAY EXCEEDS THE SIZE OF THE IN
C    $PUT ARRAY")
C
C   NXI,NYI,NXO,NYO,MX,MY EACH MUST EQUAL TWO RAISED TO SOME NON
C   NEGATIVE INTEGER POWER
    2 CALL PWRTWO(NXI,INXI)
      CALL PWRTWO(NYI,INYI)
      CALL PWRTWO(NXO,INXO)
      CALL PWRTWO(NYO,INYO)
      CALL PWRTWO(MX,IMX)
      CALL PWRTWO(MY,IMY)
C
      IF(NXO.LE.NXI.OR.NYO.LE.NYI) GO TO 220
      WRITE(6,210)
  210 FORMAT(5X,"THE SIZE OF THE FIRST FFT EXCEEDS THE SIZE OF THE INPU
     $T ARRAY")
      RETURN
C
C   MX*NXI<NXY AND MY*NYI<NXY
  220 NX=NXI*MX
      IF(NXY.GE.NX) GO TO 4
      WRITE(6,3) NXY
    3 FORMAT(I15,"  THIS DIMENSION IS INSUFFICIENT TO CARRY OUT THE MAGN
     $IFICATION IN X")
      RETURN
    4 NY=NYI*MY
      IF(NXY.GE.NY) GO TO 6
      WRITE(6,5) NXY
    5 FORMAT(I15,"  THIS DIMENSION IS INSUFFICIENT TO CARRY OUT THE MAGN
     $IFICATION IN Y")
```

```
      RETURN
C
    6 IF(NXO/2.LE.MX*MIN0(NXC-1,NXI+1-NXC)) GO TO 8
      WRITE(6,7)
    7 FORMAT(5X,"THE OUTPUT SECTOR DESIRED IS NOT CONTAINED WITHIN THE C
     $ALCULATED FFT")
      RETURN
C
    8 IF(NYO/2.LE.MY*MIN0(NYC-1,NYI+1-NYC)) GO TO 9
      WRITE(6,7)
      RETURN
C
C   DETERMINE THE ORDER IN WHICH X AND Y FFTS WILL BE CALCULATED
    9 IF(NYO.GT.NXO) GO TO 21
C
C   PERFORM SINGLE DIMENSION FFTS FOR Y THEN X
C
C LOAD Y VALUES IN XYFFT ARRAY
      NYS=NYI-NYC
      IF(NYI.EQ.1) GO TO 15
      NZ=(NY-NYI)/2
      NXYS=(NYC-1)*MY-NYO/2
      DO 14 I=1,NXI,1
      IF(NZ.EQ.0) GO TO 11
      DO 10 J=1,NZ,1
      XYFFT(J)=(0.0,0.0)
      XYFFT(J+NZ+NYI)=(0.0,0.0)
   10 CONTINUE
   11 DO 12 J=1,NYI,1
      XYFFT(J+NZ)=INPUT(I,J)
   12 CONTINUE
C
C   PERFORM SINGLE DIMENSION FFT FOR Y
      CALL PWRTWC(NY,INY)
      CALL FFTA(XYFFT,INY,ISN)
C
C   EXTRACT OUTPUT SECTOR OF INTEREST AND STORE IN INPUT ARRAY
      DO 13 J=1,NYO,1
```

```
      INPUT(I,J+NYS)=XYFFT(J+NXYS)
   13 CONTINUE
   14 CONTINUE
C
C   PERFORM NYO MAGNIFIED FFTS IN X AND STORE SELECTED  SECTORS OF FFTS
C   IN OUTPUT ARRAY
   15 NZ=(NX-NXI)/2
      NXYS=(NXC-1)*MX-NXO/2
      DO 20 J=1,NYO,1
C
C   LOAD X VALUES IN XYFFT ARRAY
      IF(NZ.EQ.0) GO TO 17
      DO 16 I=1,NZ,1
      XYFFT(I)=(0.0,0.0)
      XYFFT(I+NZ+NXI)=(0.0,0.0)
   16 CONTINUE
   17 DO 18 I=1,NXI,1
      XYFFT(I+NZ)=INPUT(I,J+NYS)
   18 CONTINUE
C
C   PERFORM SINGLE DIMENSION FFTS IN X
      CALL PWRTWC(NX,INX)
      CALL FFTA(XYFFT,INX,ISN)
C
C   EXTRACT OUTPUT SECTOR OF INTEREST AND STORE IN OUTPUT ARRAY
      DO 19 I=1,NXO,1
      OUTPUT(I,J)=XYFFT(I+NXYS)
   19 CONTINUE
   20 CONTINUE
      RETURN
C
C   PERFORM SINGLE DIMENSION FFTS FOR X THEN Y
   21 NXS= NXI-NXC
      IF(NXI.EQ.1) GO TO 27
      NZ=(NY-NXI)/2
      NXYS=(NXC-1)*MY-NXO/2
C
C   LOAD X VALUES IN XYFFT ARRAY
```

```
      DO 26 J=1,NYI,1
      IF(NZ.EQ.0) GO TO 23
      DO 22 I=1,NZ,1
      XYFFT(I)=(0.0,0.0)
      XYFFT(I+NZ+NXI)=(0.0,0.0)
   22 CONTINUE
   23 DO 24 I=1,NXI,1
      XYFFT(I+NZ)=INPUT(I,J)
   24 CONTINUE
C
C     PERFORM SINGLE DIMENSION FFT IN X
      CALL PWRTWC(NX,INX)
      CALL FFTA(XYFFT,INX,ISN)
C
C     EXTRACT OUTPUT SECTOR OF INTEREST AND STORE IN INPUT ARRAY
      DO 25 I=1,NXO,1
      INPUT(I+NXS,J)=XYFFT(I+NXYS)
   25 CONTINUE
   26 CONTINUE
C
C     PERFORM NXC MAGNIFIED FFTS IN  Y AND STORE SELECTED SECTOR OF FFT
C    IN OUTPUT ARRAY
   27 NZ=(NY-NYI)/2
      NXYS=(NYC-1)*NY-NYO/2
      DO 32 I=1,NXO,1
C
C     LOAD Y VALUES IN XYFFT ARRAY
      IF (NZ.EQ.0) GO TO 29
      DO 28 J=1,NZ,1
      XYFFT(J)= (0.0,0.0)
      XYFFT(J+NZ+NYI)=(0.0,0.0)
   28 CONTINUE
   29 DO 30 J=1,NYI,1
      XYFFT(J+NZ)=INPUT(I+NXS,J)
   30 CONTINUE
C
C     PERFORM SINGLE DIMENSION FFT IN Y
      CALL PWRTWC(NY,INY)
```

```
      CALL FFTA(XYFFT,INY,ISN)
C
C     EXTRACT OUTPUT SECTOR OF INTEREST AND STORE IN OUTPUT ARRAY
      DO 31 J=1,NYO,1
      OUTPUT(I,J)=XYFFT(J+NXYS)
   31 CONTINUE
   32 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PWRTWO(N,I)
      I=0
    1 M=2**I
      IF (N-M) 3,5,2
    2 I=I+1
      GO TO 1
    3 WRITE(6,4) N
    4 FORMAT(I15,"  THIS INTEGER DOES NOT EQUAL TWO RAISED TO A NON NEGA
     $TIVE INTEGER")
    5 RETURN
      END
```

APPENDIX A

Test Case 1 for RTFRACP

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```
TEST DATA TO TEST IBMRACP WITHOUT PLOTS (CASE I,FO,PHC,N=5)
F,F,F,F,T,00
1,2,5,16.267,3.05,15.657,48.8020001,11.80285,2.
5,.3,1,5.592,1522.,3,1,5,1
.01525,6.00,.009
.17180,2.40,.005
.01525,6.00,.009
.17180,2.40,.005
.01525,6.00,.009
3.00
0.
90.
0.
14.
16.
18.
20.
```

```
TEST DATA TO TEST IBMRACP WITHOUT PLOTS (CASE I,F0,RHC,N=5)
GRAF3D= F GRAFSA= F GRAFTR= F GRAFRV= F  TABLE= F

NFINE=    1 NPHI=  2  NTHETA=  5  GSANG=  2.00

NX,NY,NXE,NYE,NXY,MX,MY:  16  16 256   1 512  16   1
KXMAX=KYMAX=  .65094  XY SPACING=  .76812 WAVELENGTHS
 KXM=  .65094  KYM=  .04068
TANGENT OGIVE PARAMETERS:  ROS(IN)=150.46975 BOS(IN)=142.33625
                          FINOS=3.000 FINIS= 3.07245
1       RESULTS OF RADOME ANALYSIS
TEST DATA TO TEST IBMRACP WITHOUT PLOTS (CASE I,F0,RHC,N=5)
FINENESS RATIO=  3.00  DIAMETER=16.26700 IN.  LENGTH=48.80200 IN.
FREQUENCY= 11.303 GHZ
RA= 7.74700 IN.  RR=39.76878 IN.  ANTENNA D= 11.1840 WAVELENGTHS
IPOL= 3  ICASE= 1  ICPT= 1

LAYER  THICKNESS(IN.)    ER       TAND

   1        .01525     6.000     .0090
   2        .17180     2.400     .0050
   3        .01525     6.000     .0090
   4        .17180     2.400     .0050
   5        .01525     6.000     .0090


 PHI  THETA   BSEEL    BSEAZ    SLPEL      SLPAZ      GAIN
(DEG) (DEG)  (MRAD)   (MPAD) (DEG/DEG) (DEG/DEG)   (DB)


  0.0   0.0    -.00      .00   0.0000     0.0000    -2.1
  0.0  14.0    4.45    -4.82    .0182     -.0197    -1.2
  0.0  16.0    4.39    -4.23   -.0018      .0168    -1.2
  0.0  18.0    4.27    -3.71   -.0033      .0148    -1.2
  0.0  20.0    4.15    -3.27   -.0036      .0128    -1.2
 90.0   0.0     .00      .00   -.0036      .0128    -2.1
 90.0  14.0    2.97     5.61    .0122      .0270    -1.3
 90.0  16.0    2.92     5.38   -.0273     -.0067    -1.2
```

90.0	18.0	1.44	5.06	-.0167	-.0091	-1.1
90.0	20.0	1.07	5.73	-.0106	-.0094	-1.0

RECEIVED SUM VOLTAGE WITHOUT RADOME= .35400E+03

APPENDIX B

Test Case 2 for RTFRACP

```
TEST DATA TO TEST IRMRACP WITH PLOTS (CASE I,FO,RHC,N=5)
F,T,T,T,F,00
1,1,1,16.267,3.05,15.657,48.8020001,11.80285,2.
5,.3,1,5.592,1522.,3,1,5,1
.01525,6.00,.009
.17180,2.40,.005
.01525,6.00,.009
.17180,2.40,.005
.01525,6.00,.009
3.00
0.
14.
16.
18.
20.
```

```
TEST DATA TO TEST IBMRACP WITH PLOTS (CASE I,FC,RHC,N=5)
GRAF3D= F GRAFSA= T GRAFTK= T GRAFRV= T  TABLE= F

NFINE=    1 NPHI=  1  NTHETA=  1  OSANG=  2.00

NX,NY,NXF,NYE,NXY,MX,MY:  16  16 256   1 512  16   1
```

ANGLE	TPERI**2		TPARI**2		RPERI**2		RPARI**2	
.92	.819	-99.2	.819	-99.2	.156	-149.9	.156	30.1
2.07	.819	-99.2	.820	-99.2	.157	-149.8	.156	30.2
3.22	.819	-99.3	.820	-99.2	.157	-149.8	.156	30.3
4.38	.819	-99.4	.821	-99.3	.157	-149.7	.155	30.4
5.53	.818	-99.5	.821	-99.4	.158	-149.6	.154	30.6
6.69	.818	-99.6	.822	-99.5	.158	-149.4	.153	30.7
7.85	.817	-99.8	.824	-99.6	.159	-149.2	.152	31.0
9.01	.816	-100.0	.825	-99.7	.160	-149.0	.151	31.2
10.18	.815	-100.3	.826	-99.9	.160	-148.8	.149	31.6
11.35	.814	-100.5	.828	-100.1	.161	-148.6	.147	31.9
12.53	.813	-100.9	.830	-100.3	.163	-148.3	.145	32.3
13.71	.812	-101.2	.832	-100.5	.164	-148.0	.143	32.7
14.89	.810	-101.6	.834	-100.8	.165	-147.6	.141	33.2
16.09	.809	-102.0	.837	-101.0	.166	-147.3	.138	33.7
17.29	.807	-102.4	.840	-101.3	.168	-146.9	.135	34.2
18.50	.806	-102.9	.843	-101.6	.170	-146.5	.132	34.8
19.72	.804	-103.4	.846	-102.0	.171	-146.0	.129	35.4
20.94	.802	-104.0	.849	-102.3	.173	-145.6	.126	36.1
22.18	.800	-104.6	.853	-102.7	[illegible]	-145.1	.122	36.8
23.43	.798	-105.3	.857	-103.1	[illegible]	-144.6	.118	37.6
24.69	.795	-106.0	.861	-103.5	.179	-144.0	.114	38.4
25.96	.793	-106.7	.865	-104.0	.181	-143.5	.109	39.2
27.25	.791	-107.5	.870	-104.5	.184	-142.9	.105	40.1
28.55	.788	-108.4	.874	-105.0	.186	-142.3	.100	41.1
29.87	.785	-109.3	.879	-105.6	.189	-141.7	.095	42.1
31.20	.783	-110.3	.885	-106.2	.191	-141.0	.089	43.1
32.56	.780	-111.4	.890	-106.8	.194	-140.3	.084	44.2
33.93	.777	-112.5	.895	-107.5	.197	-139.7	.078	45.4

35.33	.773	-113.7	.901	-108.2	.200	-138.9	.072	46.6
36.75	.770	-115.0	.907	-108.9	.203	-138.2	.067	47.9
38.2[illegible]	.767	-116.4	.913	-109.7	.206	-137.5	.061	49.2
39.6[illegible]	.763	-117.9	.918	-110.6	.209	-136.7	.055	50.6
41.2[illegible]	.759	-119.5	.924	-111.5	.213	-136.0	.049	52.0
42.74	.755	-121.2	.930	-112.5	.216	-135.2	.043	53.5
44.33	.751	-123.1	.936	-113.6	.220	-134.4	.037	55.0
45.96	.747	-125.1	.941	-114.8	.224	-133.7	.031	56.6
47.6[illegible]	.742	-127.3	.946	-116.1	.228	-132.9	.026	58.3
49.38	.737	-129.7	.951	-117.5	.233	-132.2	.021	60.1
51.1[illegible]	.731	-132.3	.956	-119.1	.237	-131.4	.016	61.9
53.05	.725	-135.1	.960	-120.8	.243	-130.8	.012	63.6
55.01	.71[illegible]	-138.3	.963	-122.8	.249	-130.1	.009	65.8
57.07	.710	-141.[illegible]	.966	-125.0	.256	-129.6	.006	67.9
59.25	.700	-145.[illegible]	.969	-127.5	.265	-129.2	.003	70.3
61.58	.688	-150.4	.970	-130.4	.275	-128.9	.002	73.4
64.11	.672	-155.[illegible]	.971	-133.6	.290	-128.9	.001	78.5
66.88	.649	-162.2	.971	-137.8	.310	-129.3	.000	101.1
70.01	.616	-170.3	.970	-142.7	.341	-130.5	.000	-129.0
73.71	.596	178.8	.967	-149.1	.395	-133.0	.001	-116.9
78.50	.436	161.4	.959	-158.3	.514	-139.4	.001	-117.0
90.00	0.000	-180.0	0.000	-180.0	.894	-156.6	.072	-124.4

TABLE OF XMN COEFF. IS FORMED

KXMAX=KYMAX= .65094 XY SPACING= .76812 WAVELENGTHS
KXM= .65094 KYM= .04064

SUBROUTINE NORM: MIN= 0. MAX= .100E+01

SUBROUTINE NORM: MIN= 0. MAX= .100E+01

```
IPOWER OF PATTERN= 1

 SUBROUTINE NORM:  MIN=  0.          MAX=   .100E+01

 SUBROUTINE NORM:  MIN=  0.          MAX=   .100E+01

 SUBROUTINE NORM:  MIN=  0.          MAX=   .100E+01

 SUBROUTINE NORM:  MIN=  0.          MAX=   .100E+01

TANGENT OGIVE PARAMETERS:  ROS(IN)=150.46975 BOS(IN)=142.33625
                           FINOS=3.000 FINIS= 3.07245
1      RESULTS OF RADOME ANALYSIS
TEST DATA TO TEST IBMRACF WITH PLOTS (CASE I,F0,RHC,N=5)
FINENESS RATIO=  3.00  DIAMETER=16.26700 IN.  LENGTH=48.80200 IN.
FREQUENCY= 11.803 GHZ
RA= 7.7470. IN.   RR=39.7687 IN.   ANTENNA D= 11.1840 WAVELENGTHS
IPOL= 3  ICASE= 1  ICPT= 1

LAYER  THICKNESS(IN.)    ER      TAND

   1        .01525     6.000    .0090
   2        .17180     2.400    .0050
   3        .01525     6.000    .0090
   4        .17180     2.400    .0050
   5        .01525     6.000    .0090
```

```
 PHI   THETA   BSEEL    BSEAZ   SLPEL      SLPAZ      GAIN
(DEG) (DEG)   (MRAD)   (MRAD) (DEG/DEG) (DEG/DEG)   (DB)



RECEIVING PATTERN COMPUTED FOR:
  ICUT=  1
 ICOMP=  1
  KMAX=     .651
  NREC=     32
    DK=   .406844E-01
ANGMAX=  40.61
(ICUT=1 FOR EL CUT, =2 FOR AZ CUT
(ICOMP=1 FOR EL COMPONENT, =2 FOR AZIMUTH)



RECEIVING PATTERN COMPUTED FOR:
  ICUT=  1
 ICOMP=  1
  KMAX=     .651
  NREC=     32
    DK=   .406844E-01
ANGMAX=  40.61
(ICUT=1 FOR EL CUT, =2 FOR AZ CUT
(ICOMP=1 FOR EL COMPONENT, =2 FOR AZIMUTH)



MIN AND MAX VALUES OF RECV'G PATTERN:



 SUB-OUTINE NORM:  MIN=    .450E-03  MAX=    .146E+04
```

RECMG PATTERN, EL OUT, EL COMP (DB):

-39.5	-39.621	2.3
-38.[illegible]	-37.725	28.1
-36.5	-35.089	43.8
-35.1	-34.174	48.7
-33.7	-37.001	48.1
-32.3	-41.[illegible]	6.7
-30.9	-36.564	-129.0
-29.6	-31.693	-134.5
-28.2	-31.087	-140.6
-26.9	-33.941	-150.6
-25.6	-40.000	172.7
-24.3	-39.726	64.3
-23.1	-36.316	45.1
-21.8	-40.000	34.8
-20.6	-40.000	-143.9
-19.3	-30.439	-150.9
-18.1	-27.703	-155.2
-16.9	-29.545	-158.3
-15.6	-40.000	-163.7
-14.4	-28.379	19.6
-13.2	-22.832	19.5
-12.0	-22.511	21.4
-10.8	-29.557	34.4
-9.7	-25.875	-177.6
-8.5	-17.361	-167.1
-7.3	-15.087	-162.4
-6.1	-17.991	-155.1
-5.0	-26.575	-16.2
-3.8	-11.558	14.1
-2.6	-4.615	19.0
-1.5	-1.520	22.3
-.3	-.154	25.5
.9	-.175	28.9
2.0	-1.523	32.9
3.2	-4.353	38.6

4.4	-9.173	48.2
5.5	-17.241	74.3
6.7	-21.965	151.0
7.9	-13.100	-159.4
9.1	-20.644	-141.1
10.3	-27.131	-111.0
11.4	-29.660	-13.7
12.6	-24.951	22.6
13.8	-25.182	38.5
15.0	-29.936	59.3
16.2	-36.905	144.5
17.5	-30.525	-158.1
18.7	-29.242	-142.8
19.9	-32.625	-130.5
21.2	-40.000	-84.0
22.4	-37.897	30.3
23.7	-35.771	47.6
25.0	-40.000	75.8
26.3	-39.654	-161.4
27.6	-32.550	-140.9
28.9	-30.924	-134.2
30.2	-33.116	-129.7
31.6	-40.000	-113.6
33.0	-40.000	38.0
34.4	-34.909	46.0
35.8	-34.916	46.6
37.2	-38.169	37.5
38.7	[illegible]	8.0
40.2	-40.000	-9.5

MIN AND MAX VALUES OF REC'G PATTERN:

SUBROUTINE NORM: MIN= .161E-03 MAX= .736E+03

REC"G PATTERN, EL CUT, EL COMP (DB):

-39.5	-40.000	-17.5
-38.0	-35.637	-10.4
-36.5	-36.309	-1.6
-35.1	-40.000	28.2
-33.7	-40.000	95.4
-32.3	-38.291	103.4
-30.9	-40.000	50.5
-29.6	-31.969	-16.0
-28.2	-25.426	-31.2
-26.9	-22.047	-37.8
-25.6	-20.687	-42.1
-24.3	-20.852	-44.7
-23.1	-22.151	-44.8
-21.8	-23.656	-41.1
-20.5	-23.525	-37.4
-19.3	-21.048	-40.4
-18.1	-17.659	-46.9
-16.9	-14.780	-52.2
-15.6	-13.039	-55.1
-14.4	-12.690	-55.0
-13.2	-13.911	-50.9
-12.0	-16.570	-39.9
-10.8	-19.217	-23.4
-9.7	-19.064	-26.9
-8.5	-14.123	-50.7
-7.3	-8.073	-61.9
-6.1	-3.689	-64.7
-5.0	-1.040	-64.2
-3.8	-.005	-61.9
-2.6	-.660	-57.8
-1.5	-3.545	-49.6
-.3	-10.383	-20.5
.9	-9.741	76.4
2.0	-3.403	102.3
3.2	-.724	111.1
4.4	-.199	116.8

```
   5.5      -1.376     121.8
   6.7      -3.200     126.7
   7.9      -6.451     131.6
   9.1     -11.550     135.1
  10.3     -14.470     133.3
  11.4     -15.973     124.7
  12.6     -14.762     119.7
  13.8     -13.182     121.0
  15.0     -12.485     124.8
  16.2     -12.967     129.2
  17.5     -14.648     133.6
  18.7     -17.376     137.2
  19.9     -20.548     138.5
  21.2     -22.648     135.9
  22.4     -22.415     132.6
  23.7     -21.995     132.7
  25.0     -21.557     135.1
  26.3     -21.405     138.6
  27.6     -22.482     142.9
  28.9     -26.958     149.2
  30.2     -35.698     166.2
  31.6     -40.000     -99.7
  33.0     -40.000     -77.8
  34.4     -40.000    -152.0
  35.8     -37.149     160.7
  37.2     -34.461     155.7
  38.7     -36.684     157.7
  40.2     -40.000     168.2

RECEIVING PATTERN COMPUTED FOR:
  ICUT=  2
 ICOMP=  1
  KMAX=     .651
  NREC=     32
    DK=   .41534E-01
ANGMAX=  40.61
(ICUT=1 FOR EL CUT, =2 FOR AZ CUT
(ICOMP=1 FOR EL COMPONENT, =2 FOR AZIMUTH)
```

```
RECEIVING PATTERN COMPUTED FOR:
  ICUT=  2
 ICOMP=  1
  KMAX=     .651
  NREC=     32
    IK=   .-[illegible]-01
ANGMAX=  40.61
(ICUT=1 FOR EL CUT, =2 FOR AZ CUT
(ICOMP=1 FOR EL COMPONENT, =2 FOR AZIMUTH)


MIN AND MAX VALUES OF REC'G PATTERN:


 SUBROUTINE NORM:  MIN=   .571E-03  MAX=   .146E+04


REC'G PATTERN, AZ CUT,EL COMP (DB):

   -39.5      -40.000     31.5
   -38.0      -36.745     49.9
   -36.5      -33.491     59.4
   -35.1      -37.713     62.9
   -33.7      -39.957     64.0
   -32.3      -40.000   -127.1
   -30.9      -34.787   -122.0
   -29.[illegible]      -31.082   -121.7
   -28.2      -31.410   -122.9
   -26.9      -36.733   -131.5
   -25.[illegible]      -40.000    120.7
   -24.3      -37.454     77.6
   -23.1      -39.335     75.8
```

-21.8	-40.[illegible]	-150.1	343
-20.[illegible]	-32.17[illegible]	-132.3	344
-19.3	-2[illegible].767	-128.6	345
-18.1	-28.[illegible]	-126.0	346
-16.9	-36.666	-122.6	347
-15.6	-32.046	+7.8	348
-14.4	-24.978	+7.3	349
-13.2	-23.864	46.3	350
-12.[illegible]	-23.027	+8.4	351
-1[illegible].[illegible]	-31.981	-147.4	352
-9.7	-21.749	-148.9	353
-8.5	-17.851	-140.1	354
-7.3	-2[illegible].216	-138.3	355
-6.1	-31.144	4.8	356
-5.[illegible]	-12.63[illegible]	28.5	357
-3.8	-6.112	28.9	358
-2.6	-2.913	28.3	359
-1.5	-.605	27.3	360
-.3	[illegible].000	26.5	361
.9	-.617	25.9	362
2.[illegible]	-2.976	26.8	363
3.2	-6.224	27.7	364
4.4	-12.452	35.5	365
5.5	-22.048	87.6	366
6.7	-18.[illegible]	158.2	367
7.9	-15.815	171.5	368
9.1	-17.121	179.9	369
10.3	-21.[illegible]3	-165.5	370
11.[illegible]	-28.414	-125.7	371
12.6	-3[illegible].[illegible]	-58.8	372
13.8	-30.374	-17.0	373
15.[illegible]	-31.544	23.4	374
16.2	-33.2[illegible]	72.8	375
17.5	-33.[illegible]	135.4	376
18.7	-32.[illegible]31	-178.3	377
19.9	-33.385	-148.9	378
21.2	-34.613	-119.4	379
22.4	-40.[illegible]	-38.6	380

23.7	-40.000	-148.2
25.0	-40.000	-146.2
26.3	-41.000	-147.3
27.6	-36.273	-153.3
28.9	-31.584	-152.3
30.2	-31.347	-149.9
31.6	-37.585	-150.3
33.0	-40.000	45.5
34.4	-35.263	36.6
35.8	-33.397	33.4
37.2	-34.747	32.0
38.7	-37.265	30.8
40.2	-40.000	25.7

MIN AND MAX VALUES OF REC'G PATTERN:

SUBROUTINE NORM: MIN= .125E-02 MAX= .699E+03

REC'G PATTERN, AZ CUT,EL COMP (DB):

-39.5	-39.781	-10.5
-38.0	-36.254	-5.7
-36.5	-39.216	2.5
-35.1	-40.000	72.5
-33.7	-39.142	120.5
-32.3	-39.306	137.4
-30.9	-40.000	-13.7
-29.6	-29.912	-26.8
-28.2	-23.835	-28.1
-26.9	-21.102	-30.5
-25.6	-20.375	-33.3
-24.3	-21.155	-35.9
-23.1	-22.714	-39.1
-21.8	-23.300	-43.2

-20.5	-21.372	-44.7	419
-19.3	-18.037	-43.3	420
-18.1	-14.997	-42.5	421
-16.9	-13.009	-42.9	422
-15.6	-12.335	-44.1	423
-14.4	-13.077	-45.6	424
-13.2	-15.083	-47.8	425
-12.0	-17.250	-51.6	426
-10.8	-16.774	-55.0	427
-9.7	-12.809	-55.3	428
-8.5	-8.036	-55.0	429
-7.3	-4.123	-55.5	430
-6.1	-1.473	-56.4	431
-5.0	-.147	-57.4	432
-3.8	-.227	-58.3	433
-2.6	-2.054	-58.8	434
-1.5	-7.016	-57.4	435
-.3	-20.585	22.1	436
.9	-7.098	108.3	437
2.0	-2.131	110.2	438
3.2	-.356	110.4	439
4.4	-.283	110.5	440
5.5	-1.439	111.3	441
6.7	-3.632	114.1	442
7.9	-6.678	121.3	443
9.1	-9.961	134.4	444
10.3	-12.522	146.9	445
11.4	-14.364	145.5	446
12.6	-15.044	128.4	447
13.8	-14.188	113.0	448
15.0	-13.496	108.0	449
16.2	-13.404	108.8	450
17.5	-14.140	111.6	451
18.7	-15.562	116.0	452
19.9	-17.667	121.9	453
21.2	-20.314	124.7	454
22.4	-22.486	115.0	455
23.7	-22.146	101.8	456

```
   25.0       -20.919    103.5
   26.3       -20.650    113.3
   27.6       -21.936    120.8
   28.9       -25.763    121.1
   30.2       -33.923    113.7
   31.6       -40.000    -64.9
   33.0       -37.120    -77.1
   34.4       -40.000    -82.0
   35.8       -40.000    145.0
   37.2       -37.488    140.8
   38.7       -38.234    153.6
   40.2       -40.000    171.1
NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        177
NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        177
NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        177
NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        177
NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        177


FINAL ANSWERS FOR MONOPULSE SYSTEM:

K1:  -.49509E-02  .445126-02  .99998E+00
K2:  -.48156E-02  .46031E-02  .99998E+00

AZTM=  -.48156E+01 MRAD
ELTM=   .44513E+01 MRAD

MESAZ=   .27703E+00 VOLTS/DEG
MESEL=   .28484E+00 VOLTS/DEG

UAZ:  -.21485E-02  .16656E-04
UEL:   .48346E-05  .24790E-02

SMAX=   .39726299056550E+03     LCTR=   3
```

```
ADDITIONAL MONOPULSE OUTPUTS AROUND BORESIGHT:


NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        177
ANG=   -3.0 MRAD FROM BORESIGHT     VRAZ=  -.45076E-01 VOLTS     VREL=  -.47809E-01 VOLTS

DAZ(AMP,PHS)=    .56073E-01 -.54996E+02    DEL(AMP,PHS)=    .15714E+00 -.17713E+02

NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        177
ANG=   -2.0 MRAD FROM BORESIGHT     VRAZ=  -.30037E-01 VOLTS     VREL=  -.31850E-01 VOLTS

DAZ(AMP,PHS)=    .43727E-01 -.43387E+02    DEL(AMP,PHS)=    .15338E+00 -.11985E+02

NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        177
ANG=   -1.0 MRAD FROM BORESIGHT     VRAZ=  -.15009E-01 VOLTS     VREL=  -.15916E-01 VOLTS

DAZ(AMP,PHS)=    .35359E-01 -.25118E+02    DEL(AMP,PHS)=    .15128E+00 -.60391E+01

NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        177
ANG=    0.0 MRAD FROM BORESIGHT     VRAZ=   .16556E-04 VOLTS     VREL=   .48346E-05 VOLTS

DAZ(AMP,PHS)=    .32280E-01  .23564E-01    DEL(AMP,PHS)=    .15031E+00  .18355E-02

NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        177
ANG=    1.0 MRAD FROM BORESIGHT     VRAZ=   .15049E-01 VOLTS     VREL=   .15921E-01 VOLTS

DAZ(AMP,PHS)=    .35880E-01  .24798E+02    DEL(AMP,PHS)=    .15227E+00  .60018E+01

NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        177
ANG=    2.0 MRAD FROM BORESIGHT     VRAZ=   .30097E-01 VOLTS     VREL=   .31843E-01 VOLTS

DAZ(AMP,PHS)=    .44582E-01  .42461E+02    DEL(AMP,PHS)=    .15531E+00  .11831E+02

NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        177
ANG=    3.0 MRAD FROM BORESIGHT     VRAZ=   .45169E-01 VOLTS     VREL=   .47779E-01 VOLTS

DAZ(AMP,PHS)=    .56280E-01  .53653E+02    DEL(AMP,PHS)=    .15996E+00  .17379E+02
```

```
AVERAGE SLPAZ=    .26249E+00 VOLTS/DEG
AVERAGE SLPEL=    .27823E+00 VOLTS/DEG
 SUM=1.0 VOLT

 0.0  14.0    4.45   -4.82   0.0000    0.0000   -1.2

RECEIVED SUM VOLTAGE WITHOUT RADOME=  .35400E+03
```

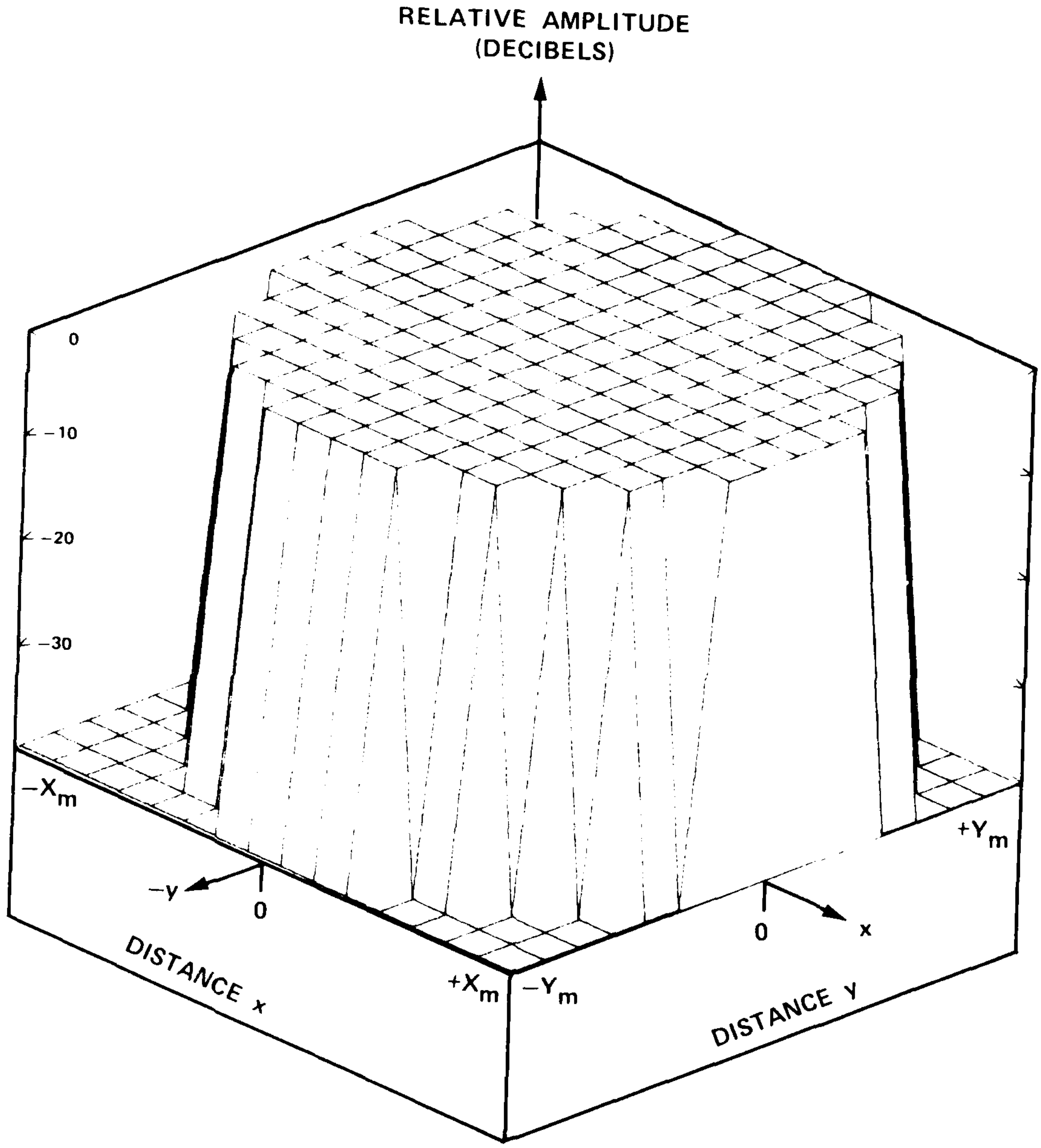

Figure B-1. $|E_{x\Sigma}|$ or $|E_{y\Sigma}|$ of the RHC (ICASE=1) Antenna.

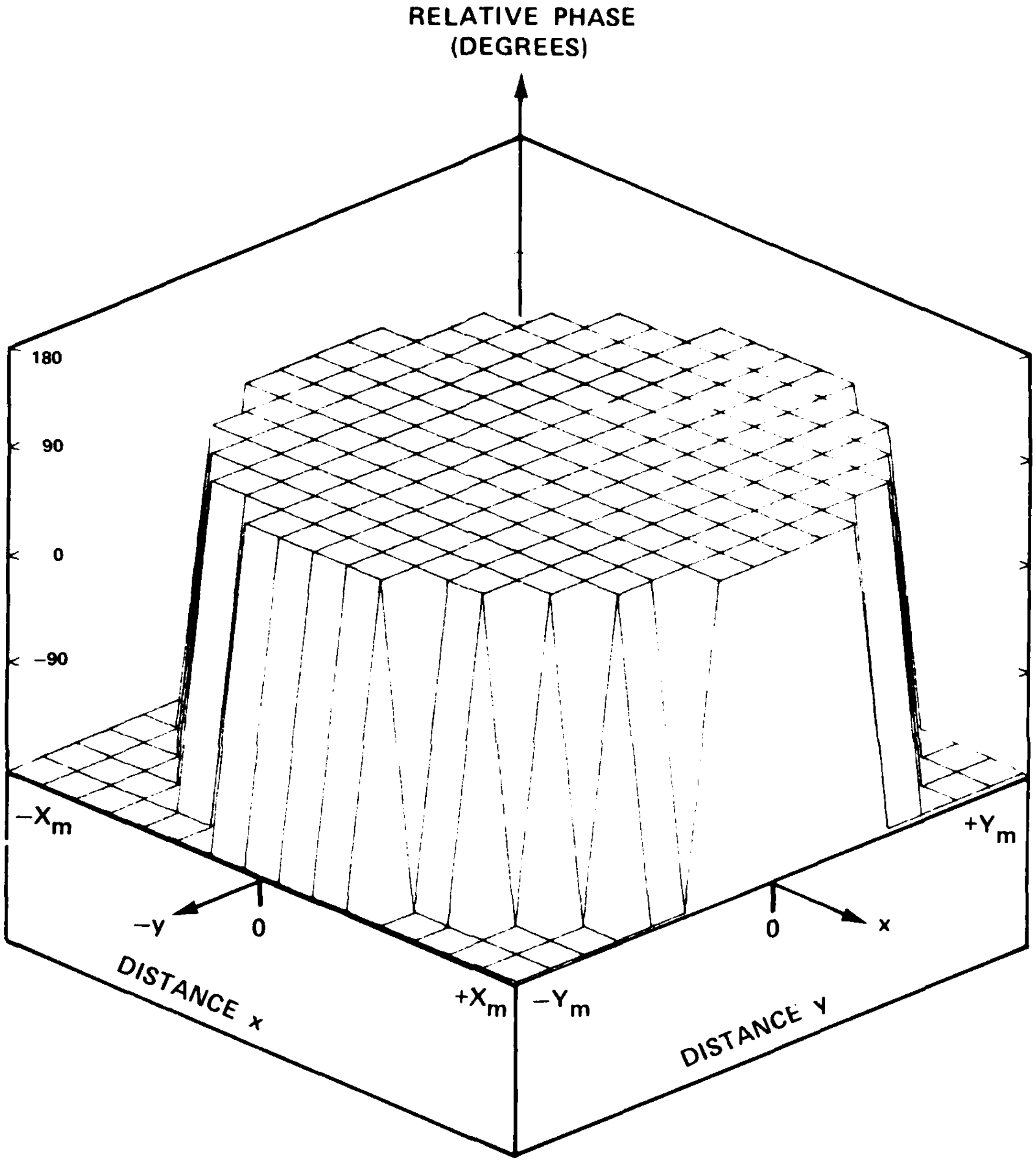

Figure B-2. Phase of $E_{x\Sigma}$ for RHC (ICASE=1) Antenna.

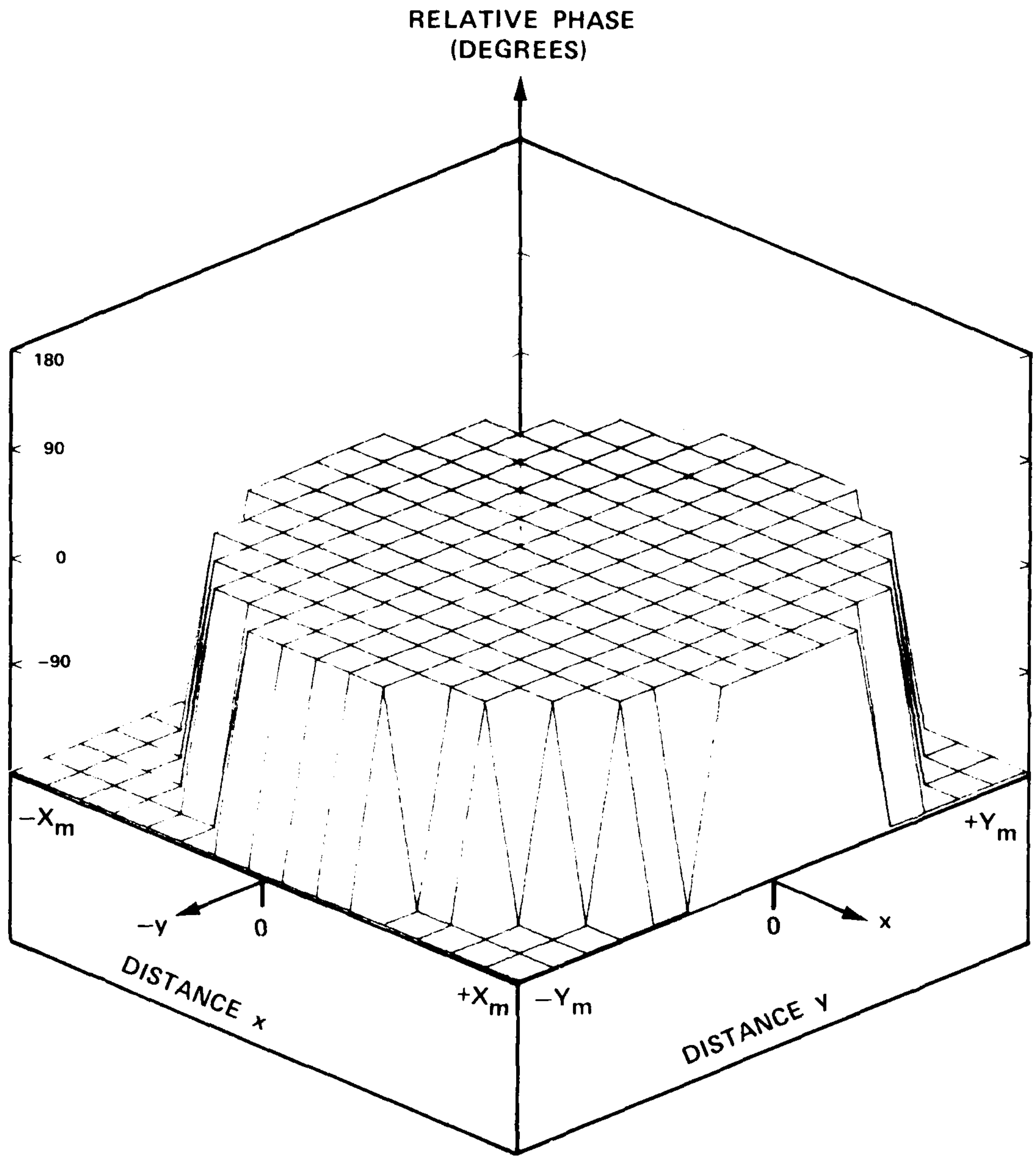

Figure B-3. Phase of $E_{y\Sigma}$ for RHC Antenna.

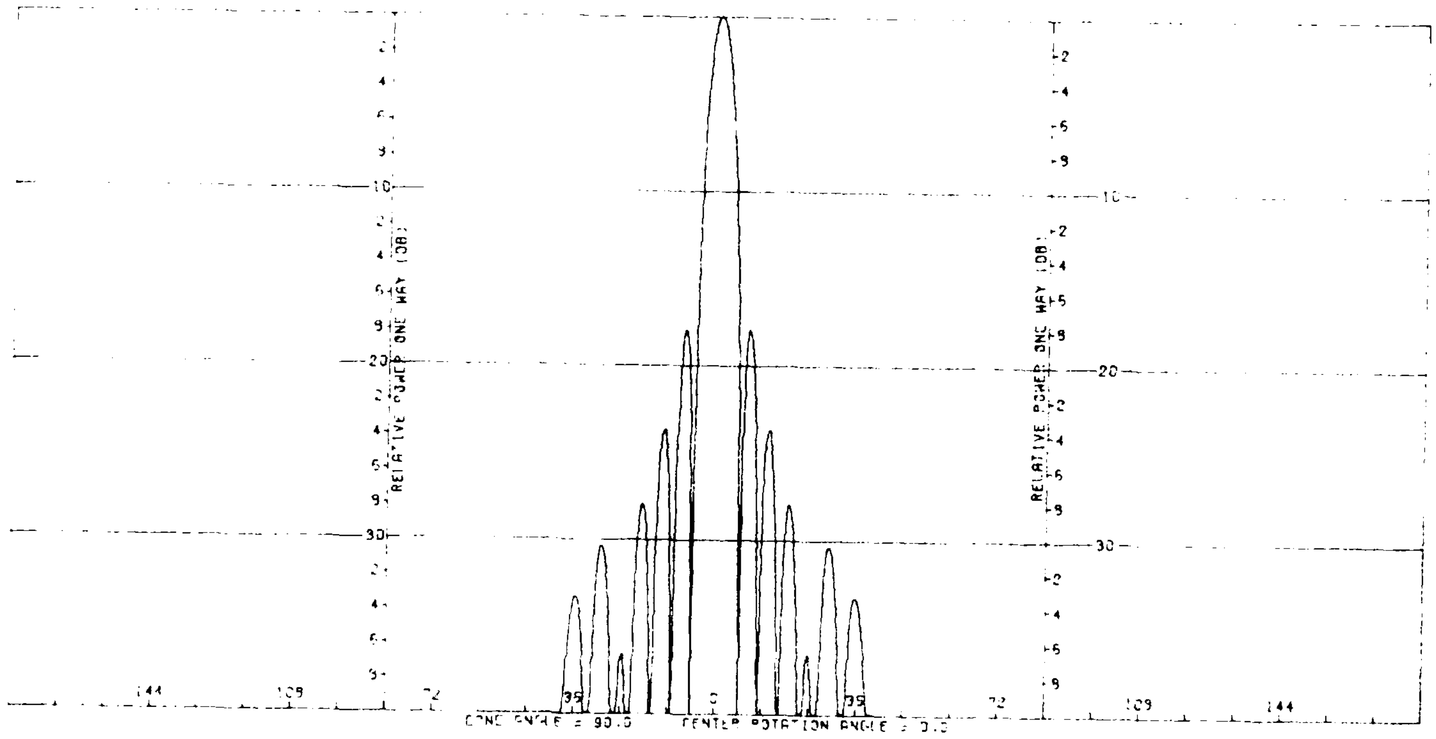

Figure B-4. Transmitting E-Plane Σ Pattern of RHC Antenna Without Radome.

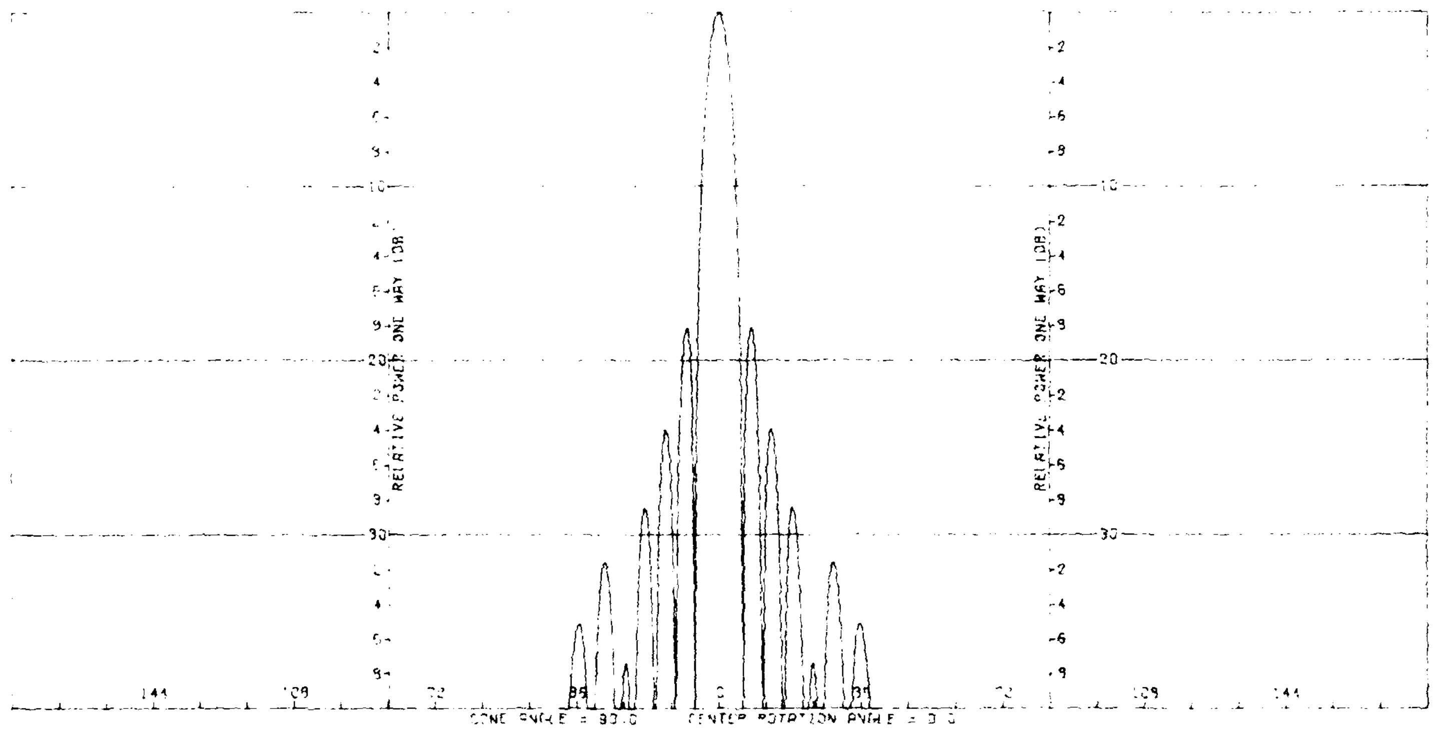

Figure B-5. Transmitting H-Plane Pattern of RHC Antenna Without Radome.

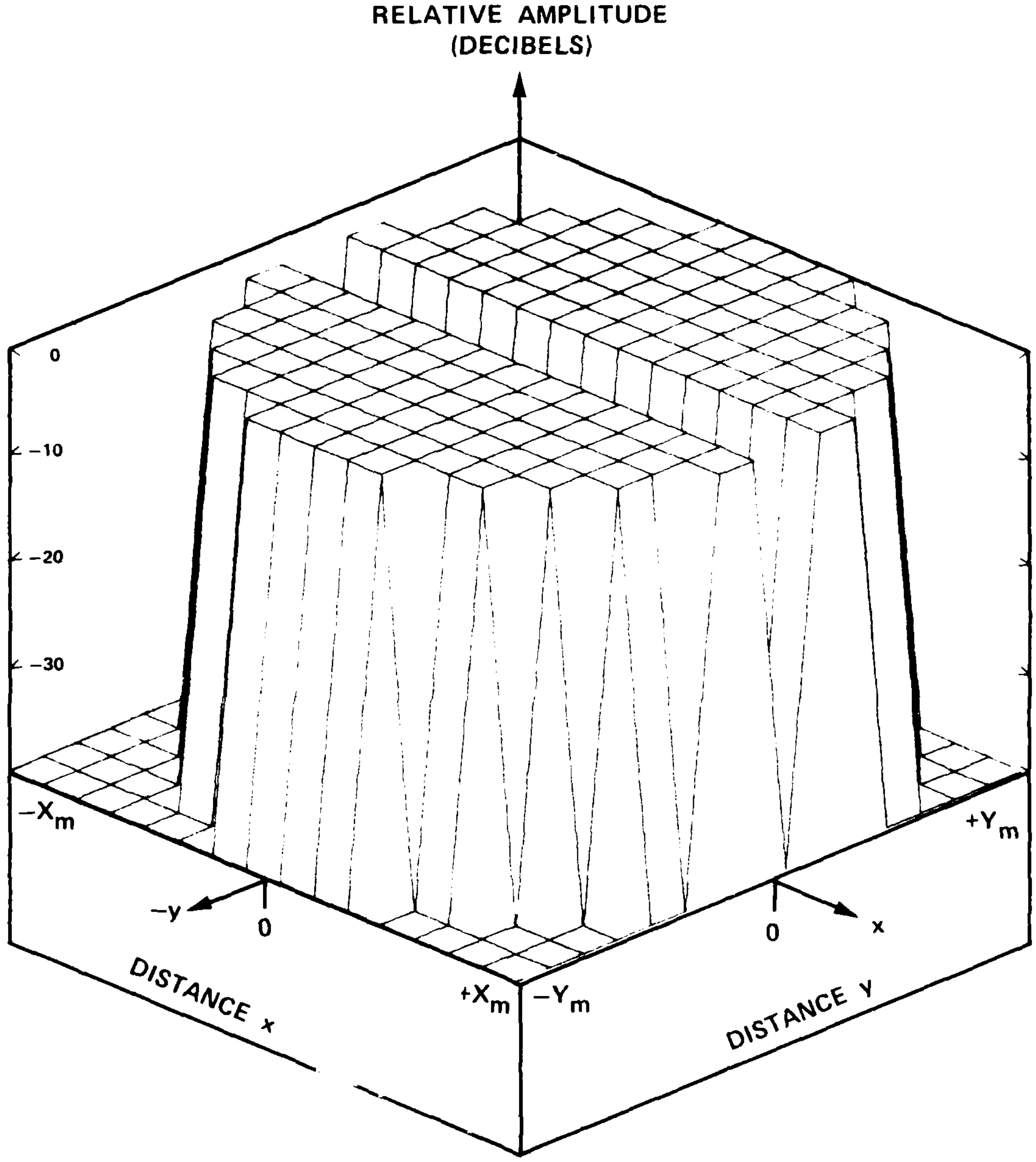

Figure B-6. $|E_x|_{\Delta EL}$ or $|E_y|_{\Delta EL}$ of RHC Antenna.

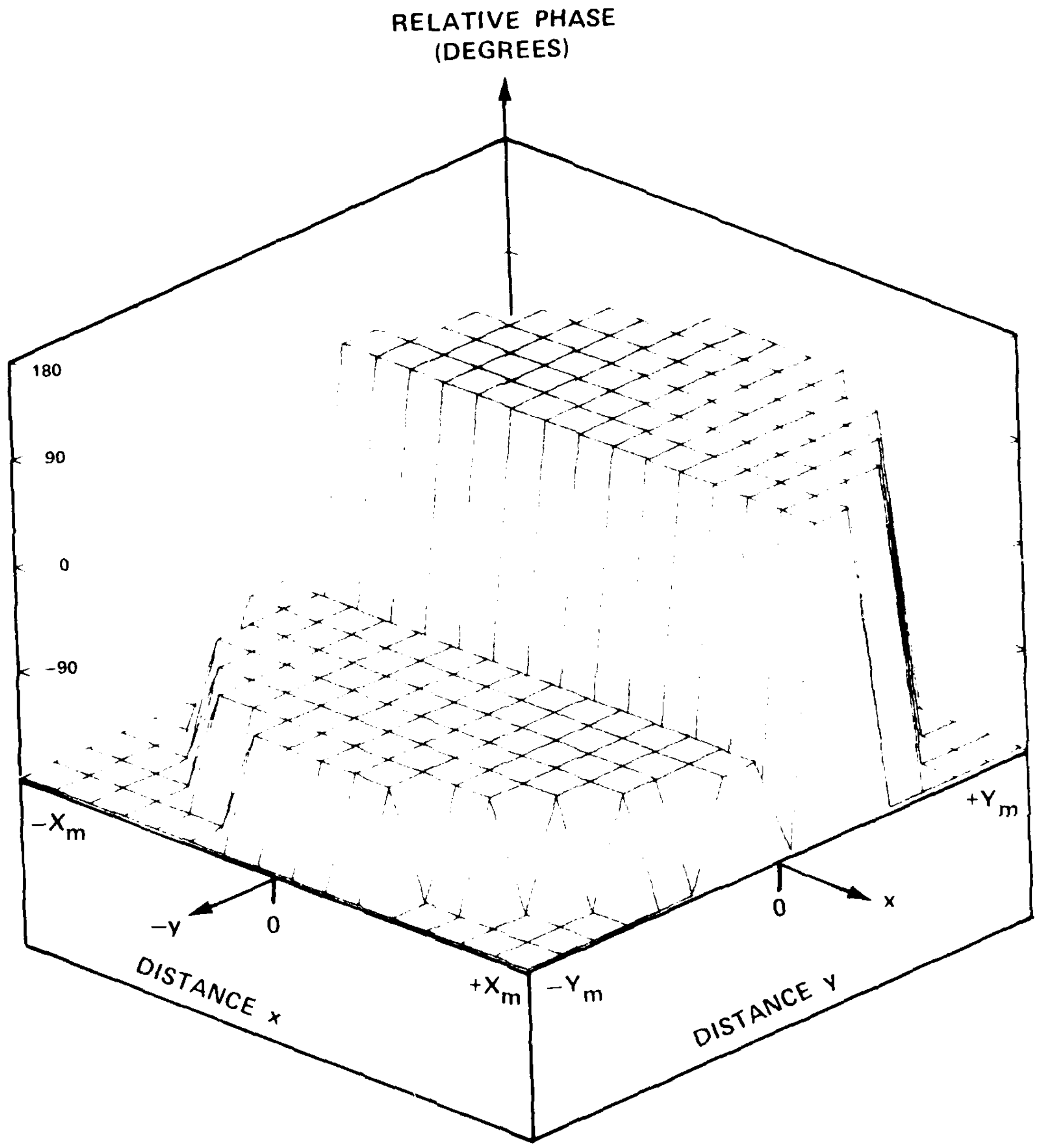

Figure B-7. Phase of E_{xAEL} of RHC Antenna.

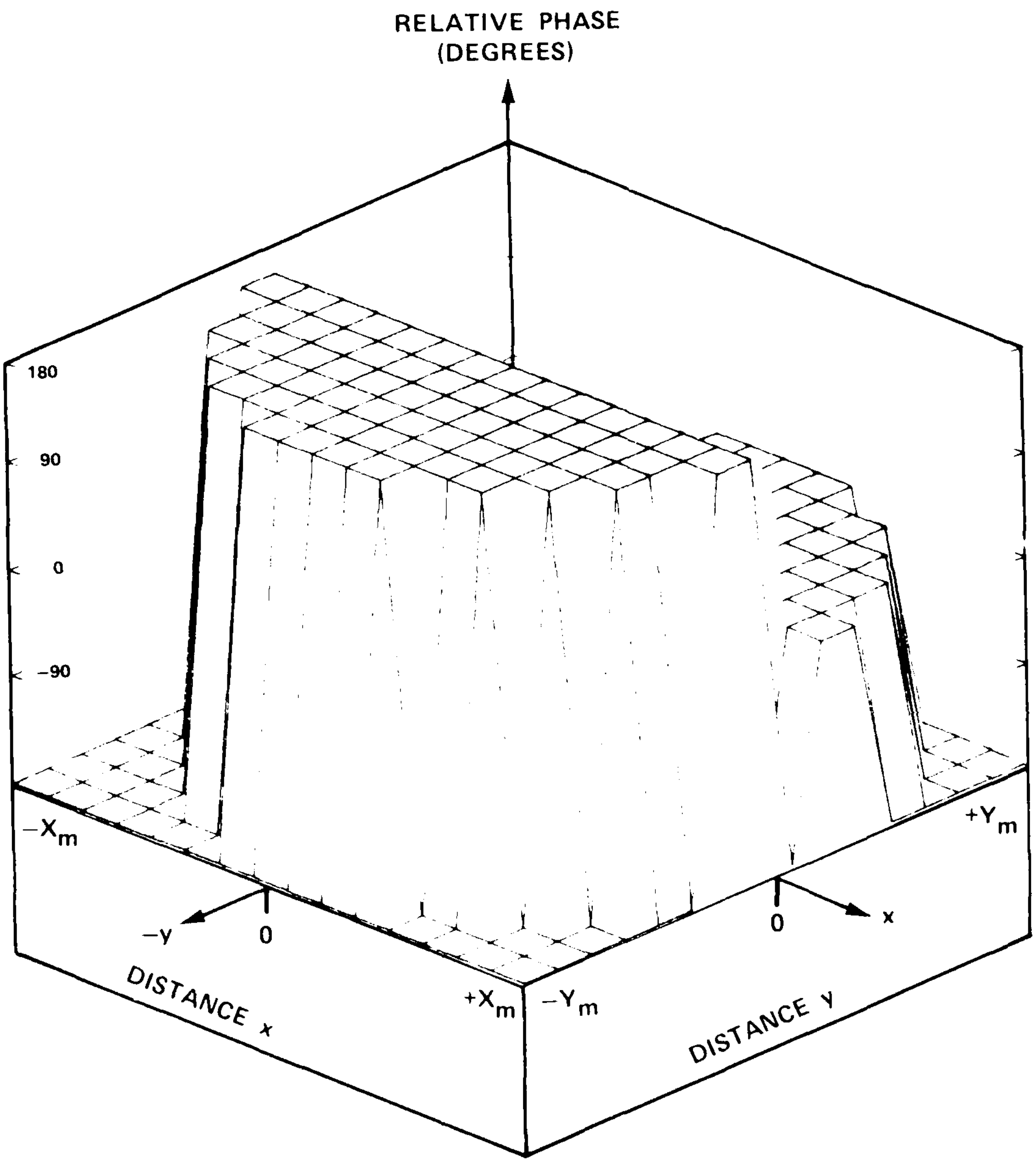

Figure B-8. Phase of $E_{y\Lambda EL}$ of RHC Antenna.

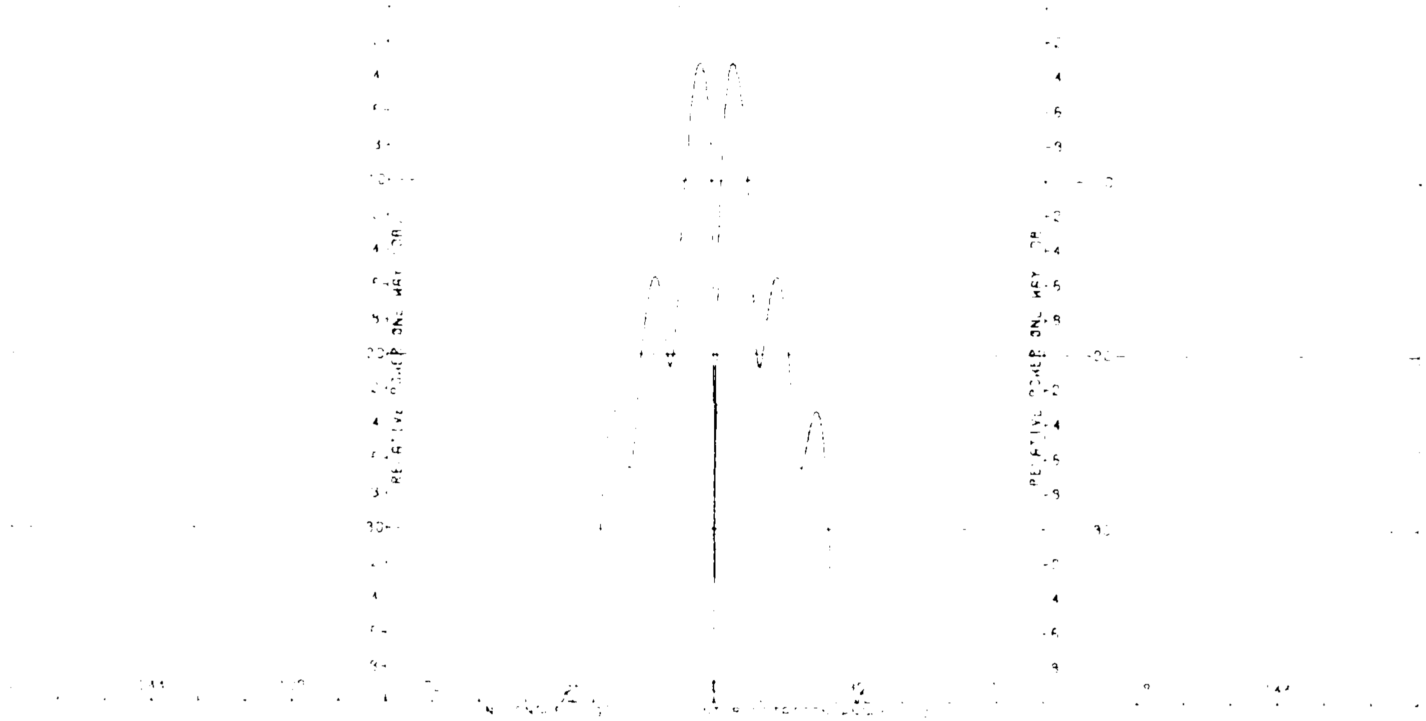

Figure B-[illegible]. Transmitting E-Plane P_{EL} Pattern [illegible] Antenna With [illegible] Radome.

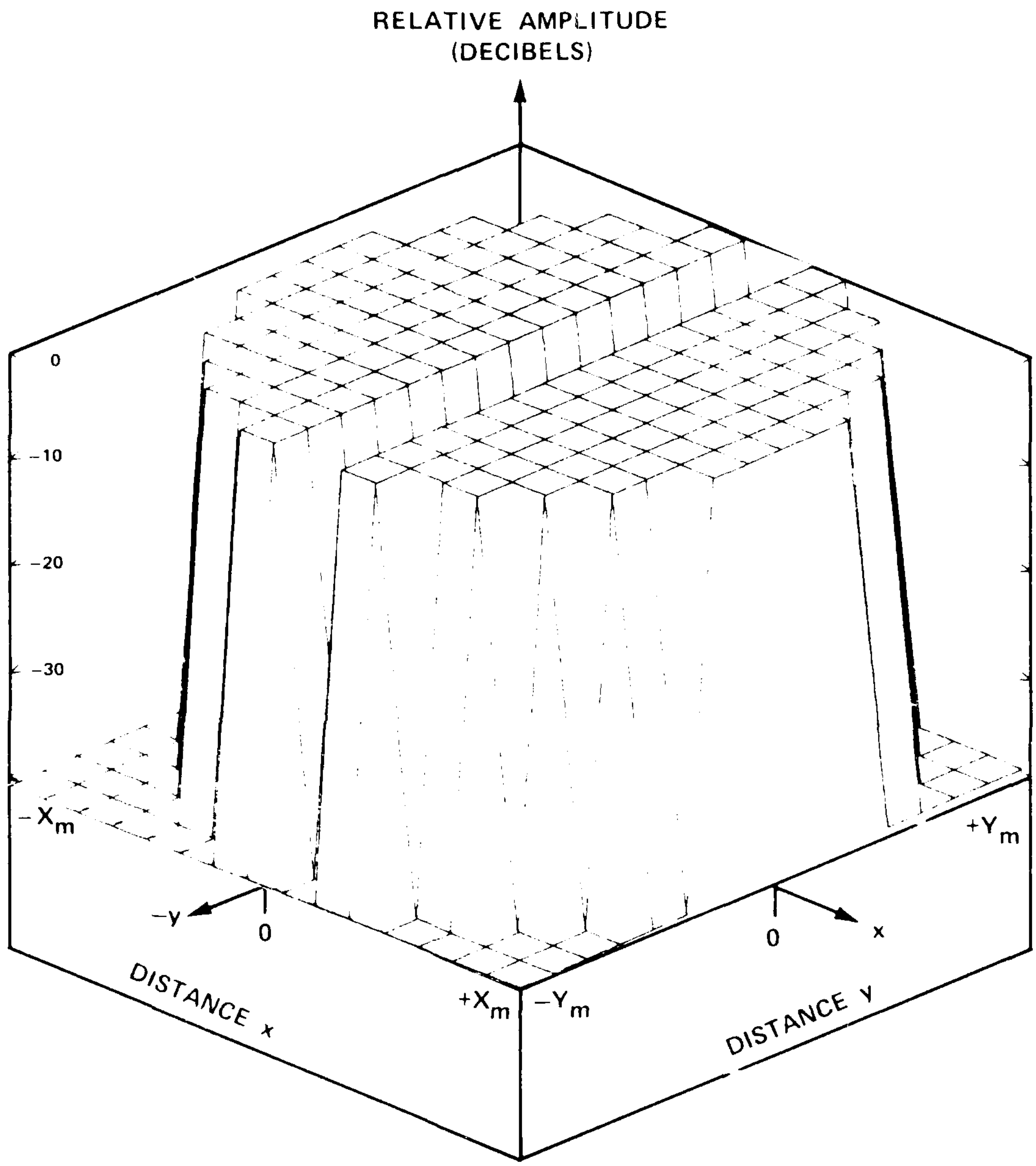

Figure B-10. $|E_x|_{\Delta AZ}$ or $|E_y|_{\Delta AZ}$ of RHC Antenna.

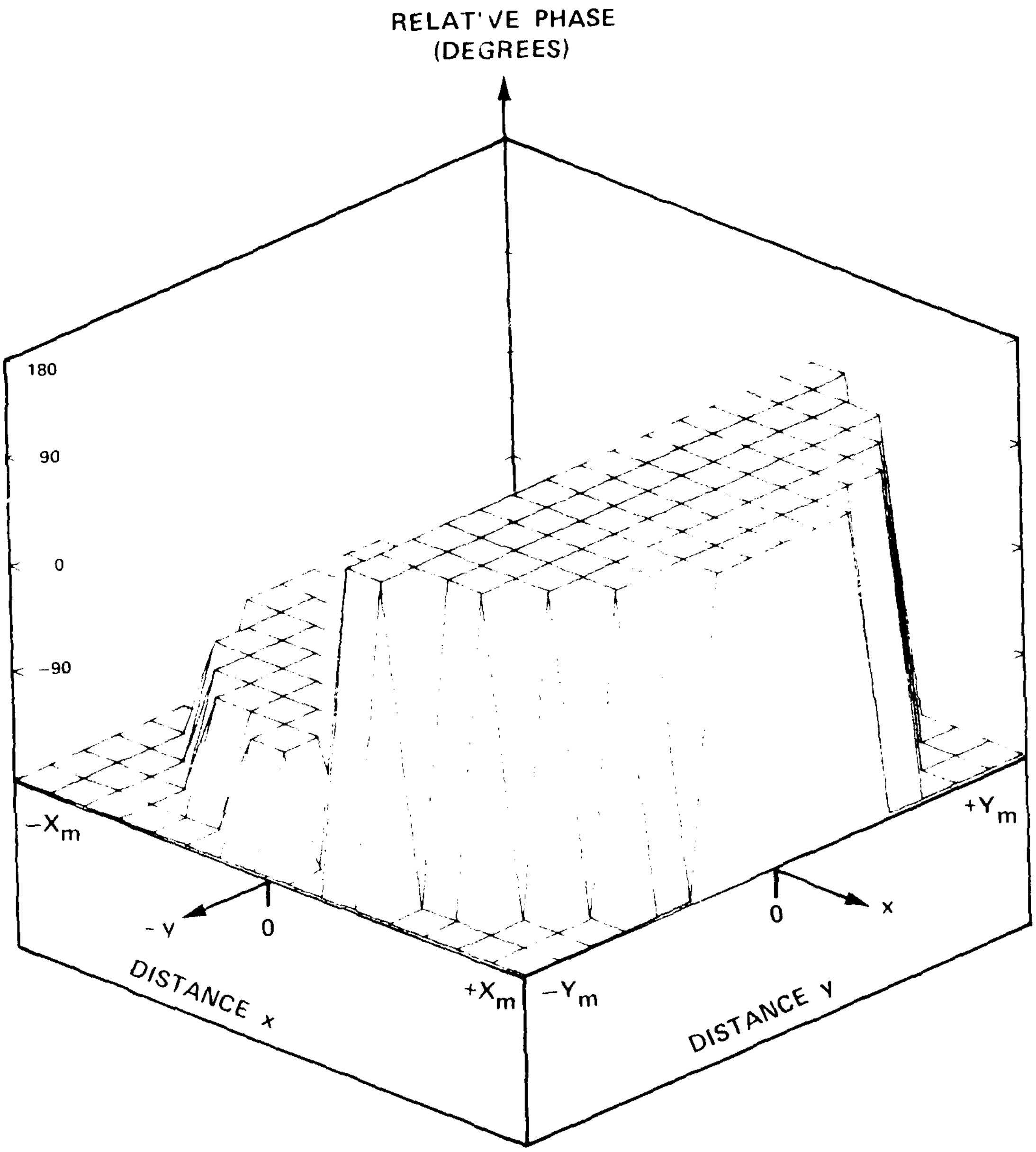

Figure B-11. Phase of $E_{x\Delta AZ}$ of RHC Antenna.

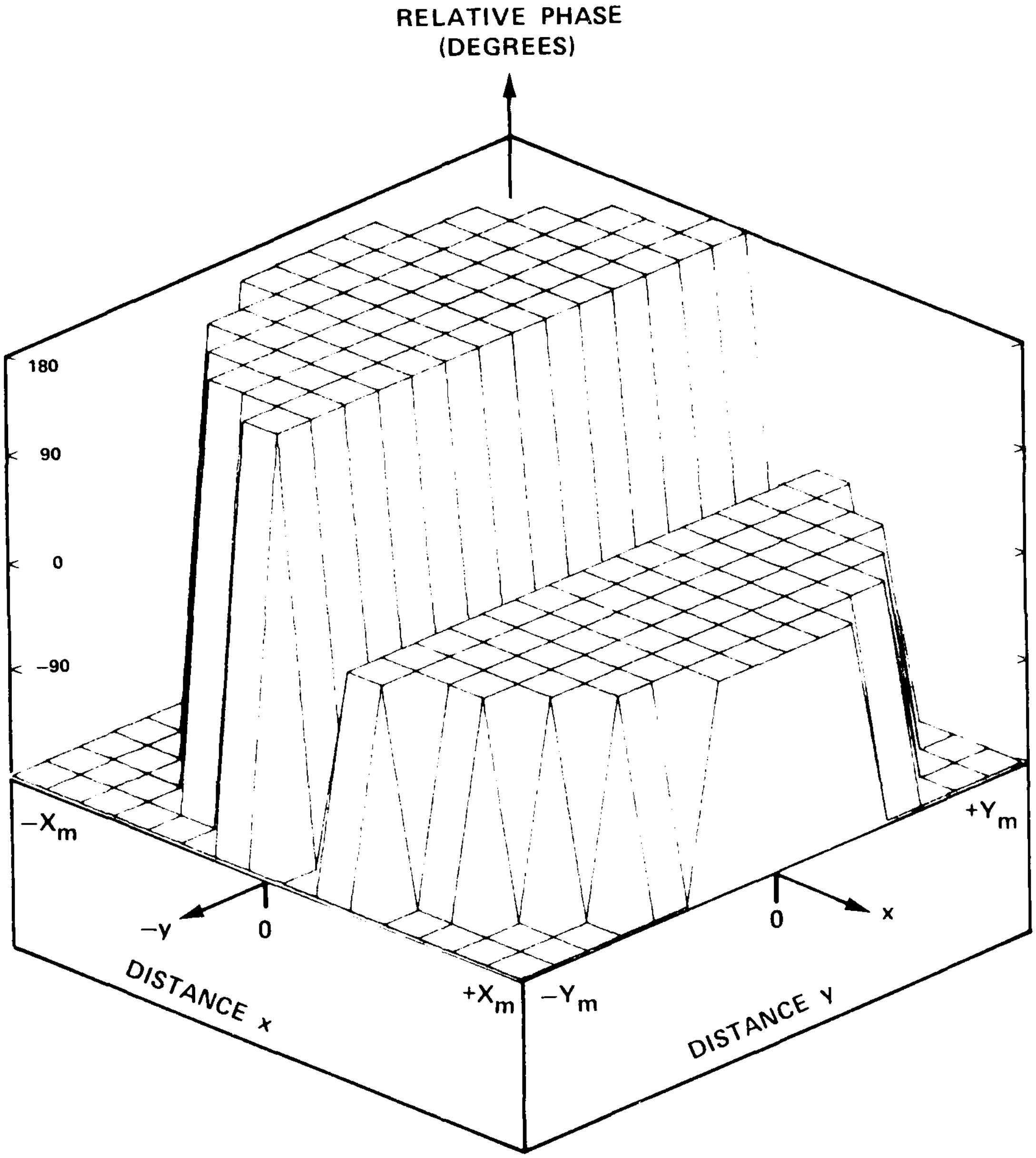

Figure B-12. Phase of $E_{y\Delta AZ}$ of RHC Antenna.

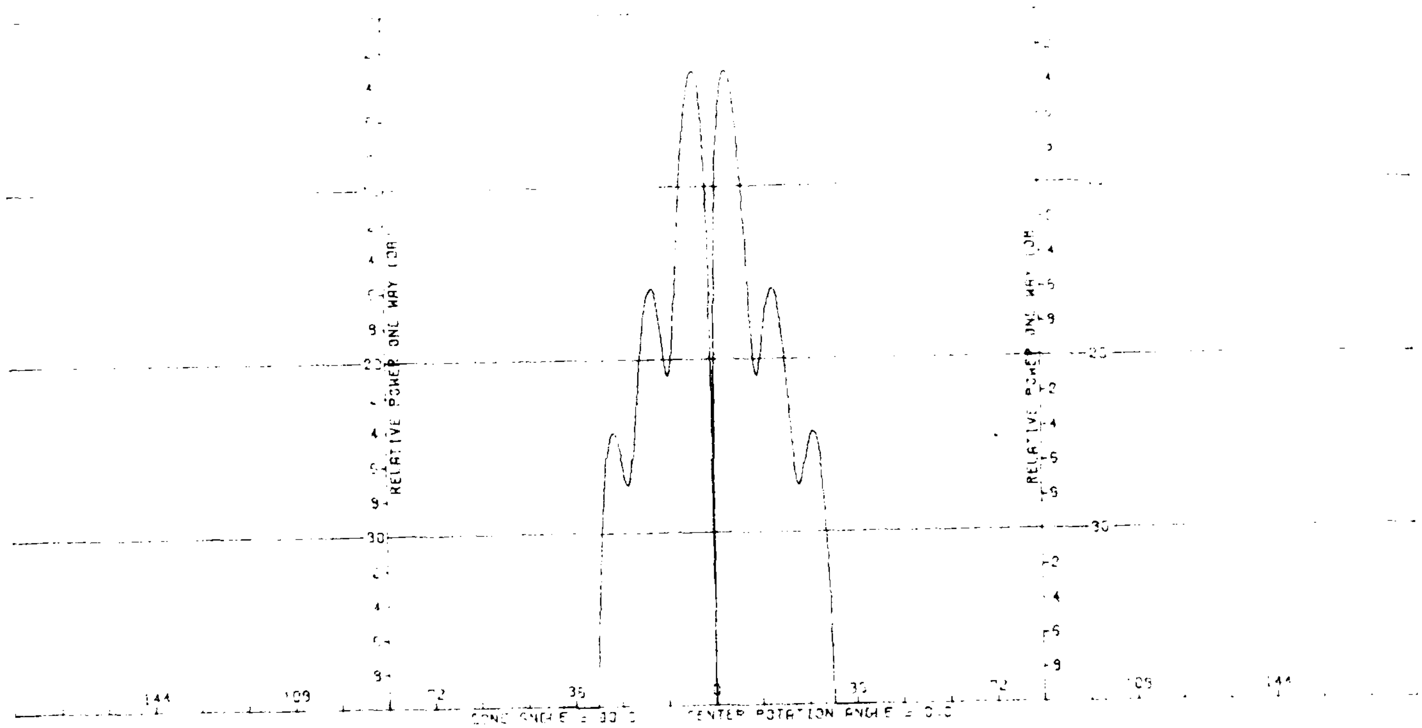

Figure B-13. Transmitting H-Plane Δ_{AZ} Pattern of RHC Antenna Without Radome.

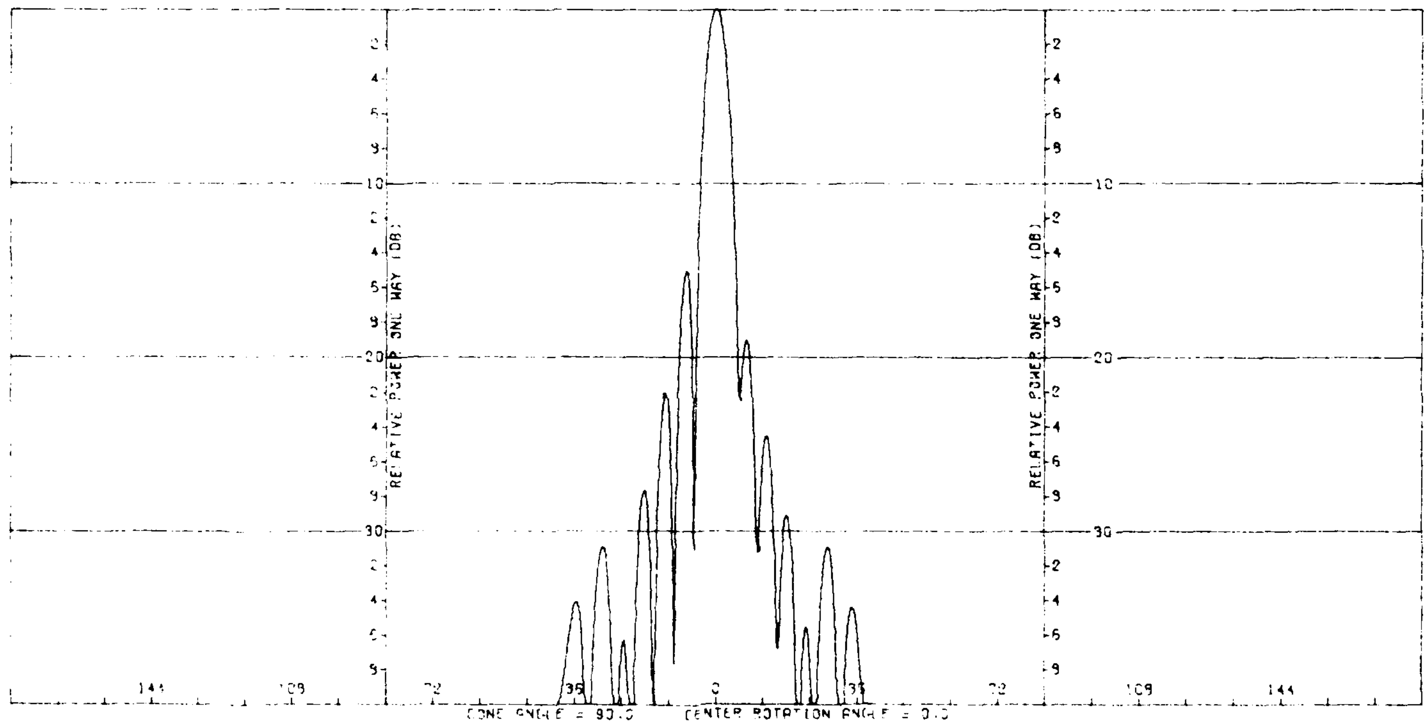

Figure B-14. Receiving E-Plane Σ Pattern of RHC Antenna With Radome at (0°,14°).

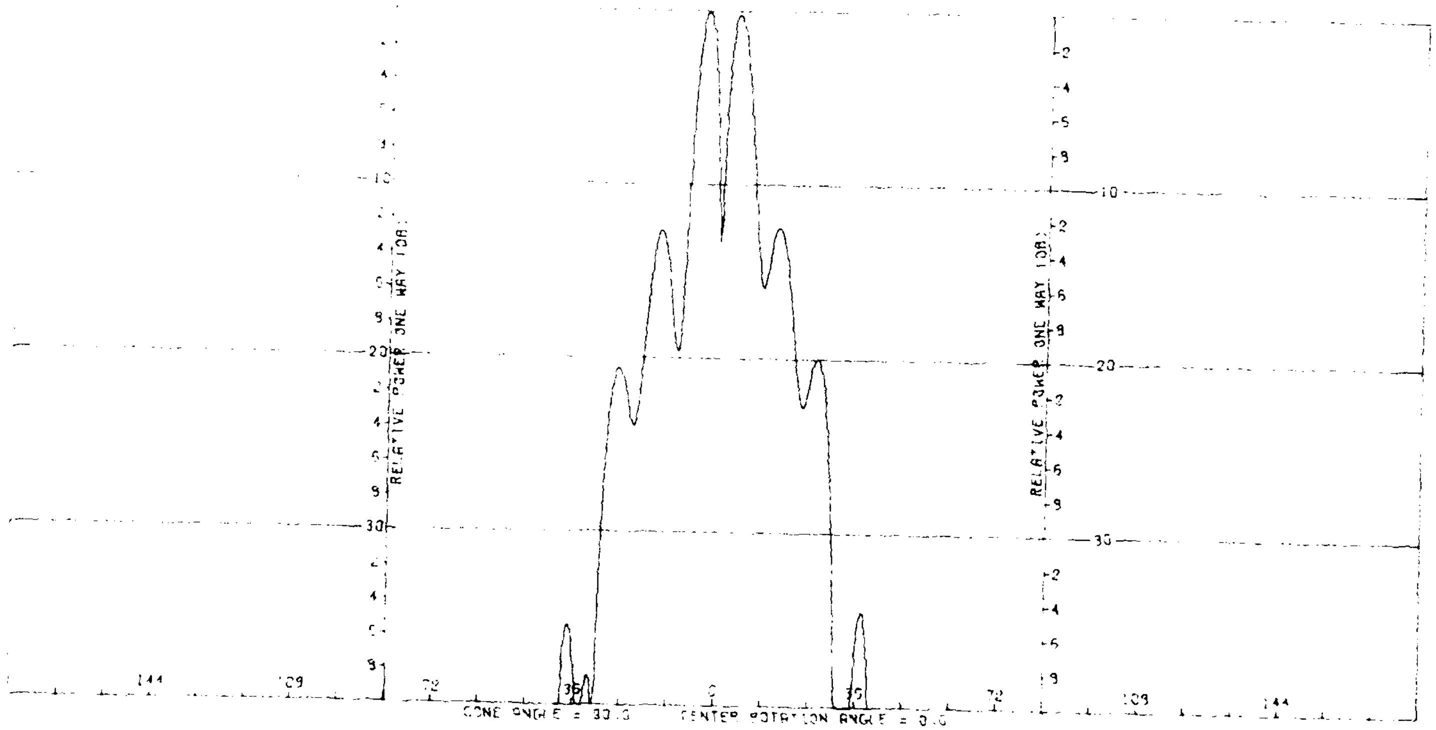

Figure B-15. Receiving E-Plane A_{EL} Pattern of RHC Antenna With Radome at (0°,14°).

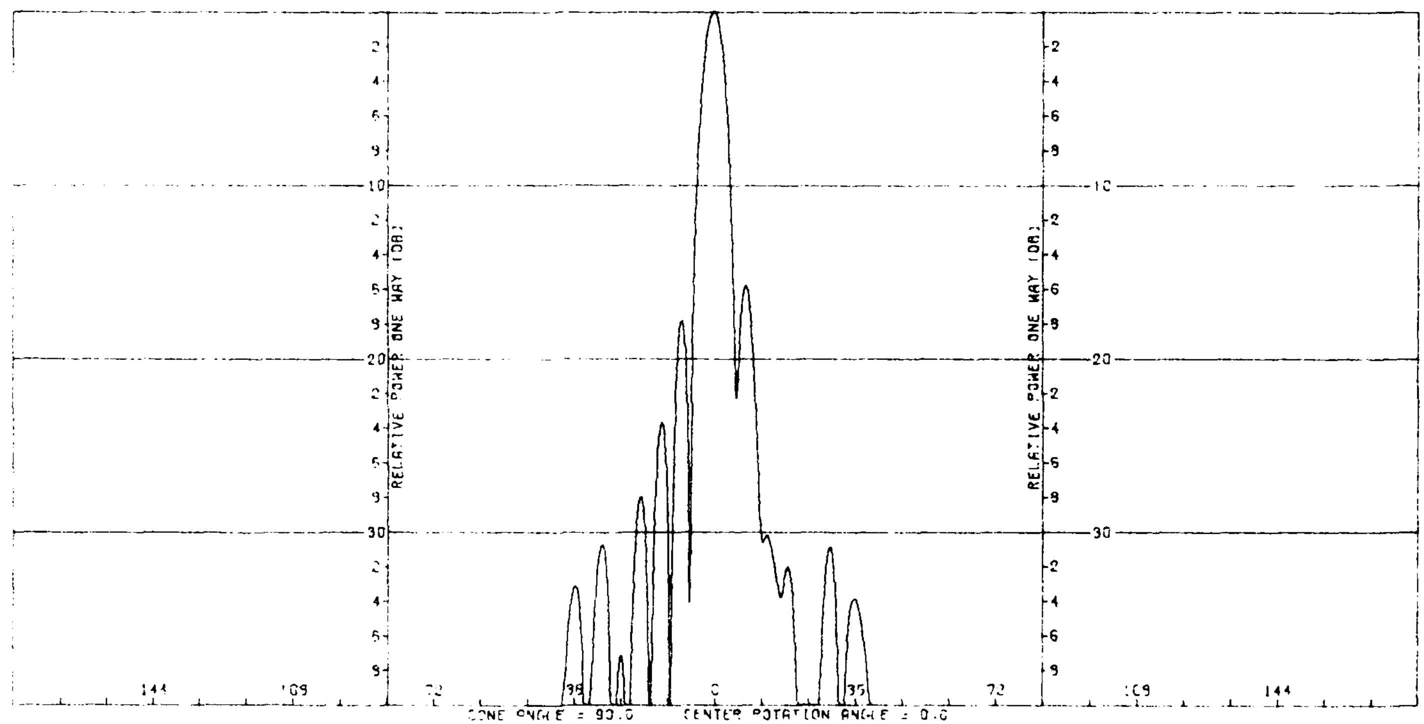

Figure B-16. Receiving H-Plane Σ Pattern of RHC Antenna With Radome at (0°,14°).

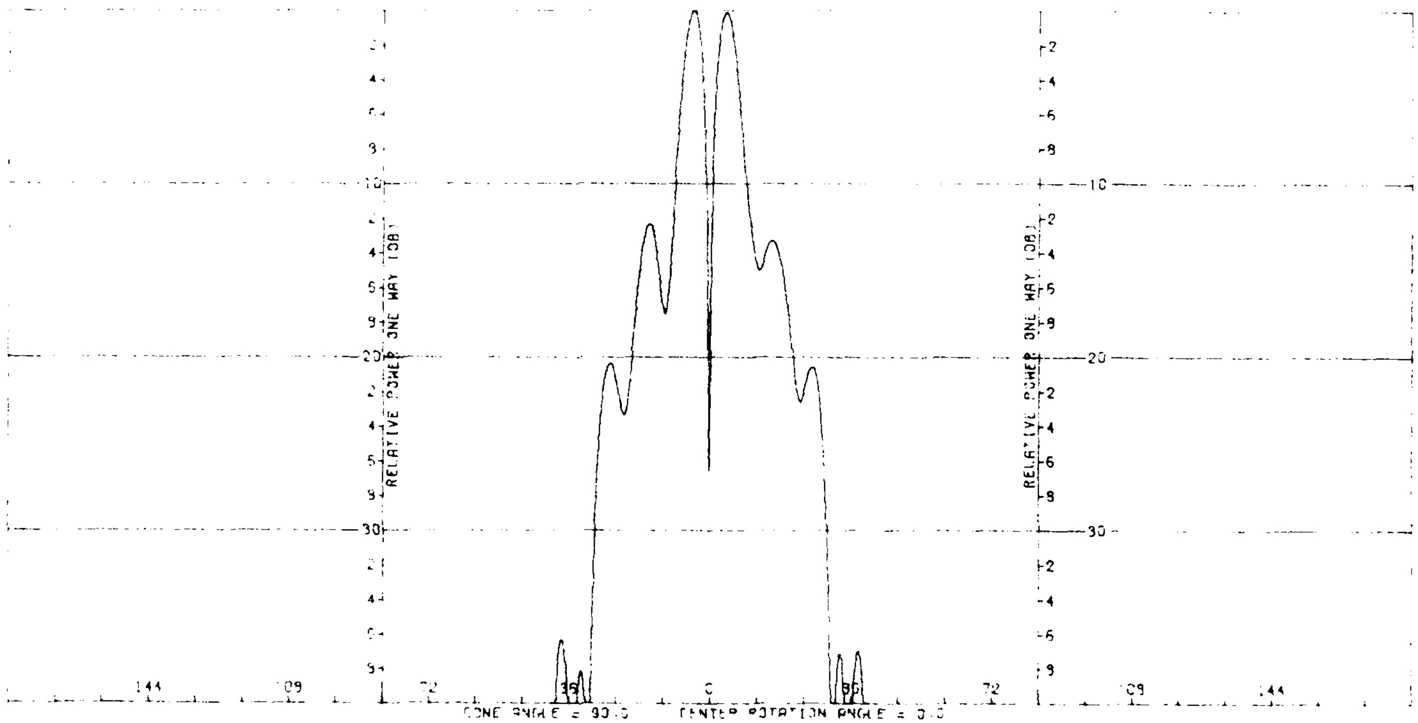

Figure B-17. Receiving H-Plane Δ_{AZ} Pattern of RHC Antenna With Radome at (0°,14°).

APPENDIX C

Test Case 3 for RTFRACP

```
TEST DATA TO TEST IRMRACP WITHOUT PLOTS (CASE 3,FO,LINEAR,N=5)
F,F,F,F,T,03
1,2,5,16.267,3.05,15.657,48.8020001,11.80285,2.
5,.3,1,2.5996,1522.,1,3,5,1
.01525,6.00,.009
.17180,2.40,.005
.01525,6.00,.009
.17180,2.40,.005
.01525,6.00,.009
3.00
0.
90.
0.
14.
16.
18.
20.
```

```
TEST DATA TO TEST IBMRACP WITHOUT PLOTS (CASE 3,F0,LINEAR,N=5)
GRAFMU= F GRAFDA= F GRAFTR= F GRAFRV= F  TABLE= F

NFIN = 1 NPHI= 2 NTHETA= 5 OSANG= 2.00

NX,NY,NXF,NYF,NXY,MX,MY: 16 16 256 1 512 16 1
KXMAX=KYMAX= 1.38696  XY SPACING= .36050 WAVELENGTHS
 KXM= 1.38696  KYM= .08669
TANGENT OGIVE PARAMETERS:  RUS(IN)=150.45975 BOS(IN)=142.33625
                           FINOS=3.000 FINIS= 3.07245
1      RESULTS OF RADOME ANALYSIS
TEST DATA TO TEST IBMRACP WITHOUT PLOTS (CASE 3,F0,LINEAR,N=5)
FINENESS RATIO= 3.00  DIAMETER=16.26700 IN.  LENGTH=48.80200 IN.
FREQUENCY= 11.803 GHZ
RA= 7.74700 IN.  RP=33.76373 IN.  ANTENNA D= 5.1992 WAVELENGTHS
IPOL= 1  ICASE= 3  IOPT= 1

LAYER  THICKNESS(IN.)   ER      TAND

  1       .01525      6.000    .0090
  2       .17180      2.400    .0050
  3       .01525      6.000    .0090
  4       .17180      2.400    .0050
  5       .01525      6.000    .0090


 PHI   THETA   BSEEL    BSEAZ    SLPEL      SLPAZ     GAIN
(DEG)  (DEG)   (MRAD)   (MRAD)  (DEG/DEG)  (DEG/DEG)  (DB)


  0.0    0.0     .00     -.00    0.0000     0.0000    -1.6
  0.0   14.0     .00    -5.37     .0003     -.0220    -1.6
  0.0   16.0     .00    -4.77    -.0000      .0170    -1.6
  0.0   14.0     .00    -4.27    -.0000      .0143    -1.5
  0.0   20.0     .00    -3.84    -.0000      .0123    -1.5
 90.0    0.0     .00     -.00    -.0000      .0123    -1.5
 90.0   14.0   -2.27     -.00    -.0093     -.0000     -.3
 90.0   16.0   -2.19     -.00     .0026      .0000     -.3
```

[illegible]	[illegible]	-[illegible]	-.[illegible]	.[illegible]37	.000C	-.2	33
[illegible]	[illegible]	-1.9[illegible]	-.[illegible]	.0[illegible]43	.0000	-.2	+[illegible]
							+1
							+2
							+3
							44
							+5

[illegible] SUM VOLTAGE WITHOUT RADOME= .14615E+03

APPENDIX D

Test Case 4 for RTFRACP

```
TEST DATA TO TEST IPMRACP WITH PLOTS (CASE 3,F2,LINEAR,N=5)
F,T,T,T,F,00
1.1.1.16.267,3.05,15.657,48.802000,1.11.80285,2.
5.,3,1,2.5996,1522.,1,3,5,1
.01525,6.00,.009
.17180,2.40,.005
.01525,6.00,.009
.17180,2.40,.005
.01525,6.00,.009
3.00
0.
14.
16.
19.
20.
```

```
TEST DATA TO TEST IBMRACP WITH PLOTS (CASE 3,FJ,LINEAR,N=5)
GRAFSD= F GRAFSA= T GRAFTP= T GRAFRV= T   TABLF= F

NFINE=     1 NPHI=  1  NTHETA=  1  OSANG=   2.00

NX,NY,NXF,NYE,NXY,MX,MY=  16   16 256    1 512  16    1
```

ANGLE	TPERI**2		TPARI**2		RPERI**2		RPARI**2	
.92	.813	-99.2	.819	-99.2	.156	-149.9	.156	30.1
2.07	.819	-99.2	.820	-99.2	.157	-149.8	.156	30.2
3.22	.819	-99.3	.820	-99.2	.157	-149.8	.156	30.3
4.38	.819	-99.4	.821	-99.3	.157	-149.7	.155	30.4
5.53	.818	-99.5	.821	-99.4	.158	-149.6	.154	30.6
6.69	.818	-99.6	.822	-99.5	.158	-149.4	.153	30.7
7.85	.817	-99.8	.824	-99.6	.159	-149.2	.152	31.0
9.01	.816	-100.0	.825	-99.7	.160	-149.0	.151	31.2
10.16	.815	-100.3	.826	-99.9	.160	-148.8	.149	31.6
11.35	.814	-100.5	.828	-100.1	.161	-148.6	.147	31.9
12.53	.813	-100.9	.830	-100.3	.163	-148.3	.145	32.3
13.71	.812	-101.2	.832	-100.5	.164	-148.0	.143	32.7
14.89	.810	-101.6	.834	-100.8	.165	-147.6	.141	33.2
16.09	.809	-102.0	.837	-101.0	.166	-147.3	.138	33.7
17.29	.807	-102.4	.840	-101.3	.168	-146.9	.135	34.2
18.50	.806	-102.9	.843	-101.6	.170	-146.5	.132	34.8
19.72	.804	-103.4	.846	-102.0	.171	-146.0	.129	35.4
20.94	.802	-104.0	.849	-102.3	.173	-145.6	.126	36.1
22.18	.800	-104.6	.853	-102.7	.175	-145.1	.122	36.9
23.43	.798	-105.3	.857	-103.1	.177	-144.6	.118	37.6
24.69	.795	-106.0	.861	-103.5	.179	-144.0	.114	38.4
25.96	.793	-106.7	.865	-104.0	.181	-143.5	.109	39.2
27.25	.791	-107.5	.870	-104.5	.184	-142.9	.105	40.1
28.55	.788	-108.4	.874	-105.0	.186	-142.3	.100	41.1
29.87	.785	-109.3	.879	-105.6	.189	-141.7	.095	42.1
31.20	.783	-110.3	.885	-106.2	.191	-141.0	.089	43.1
32.56	.780	-111.4	.890	-106.8	.194	-140.3	.084	44.2
33.9[illegible]	.777	-112.5	.895	-107.5	.197	-139.7	.078	45.4

AD-A099 182 GEORGIA INST OF TECH ATLANTA F/G 17/9
PARAMETRIC INVESTIGATION OF RADOME ANALYSIS METHODS. VOLUME II.--ETC(U)
FEB 81 G K HUDDLESTON, H L BASSETT AFOSR-77-3469
UNCLASSIFIED AFOSR-TR-81-0460 NL

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35.33	.773	-113.7	.901	-108.2	.200	-138.9	.072	46.6
36.75	.770	-115.0	.907	-108.9	.203	-138.2	.057	47.9
38.20	.767	-116.4	.913	-109.7	.206	-137.5	.051	49.2
39.68	.763	-117.9	.918	-110.6	.209	-136.7	.055	50.6
41.20	.759	-119.5	.924	-111.5	.213	-136.0	.049	52.0
42.74	.755	-121.2	.930	-112.5	.216	-135.2	.043	53.5
44.33	.751	-123.1	.936	-113.6	.220	-134.4	.037	55.0
45.96	.747	-125.1	.941	-114.8	.224	-133.7	.031	56.6
47.04	.742	-127.3	.946	-116.1	.228	-132.9	.026	58.3
49.38	.737	-129.7	.951	-117.5	.233	-132.2	.021	60.1
51.18	.731	-132.3	.956	-119.1	.237	-131.4	.016	61.9
53.05	.725	-135.1	.960	-120.8	.243	-130.8	.012	63.8
55.01	.718	-138.3	.963	-122.8	.249	-130.1	.009	65.8
57.07	.710	-141.9	.966	-125.0	.256	-129.6	.006	67.9
59.25	.700	-145.9	.969	-127.5	.265	-129.2	.003	70.3
61.58	.688	-150.4	.970	-130.4	.275	-128.9	.002	73.4
64.11	.672	-155.8	.971	-133.8	.290	-128.9	.001	78.5
66.88	.649	-162.2	.971	-137.8	.310	-129.3	.000	101.1
70.01	.616	-170.3	.970	-142.7	.341	-130.5	.000	-128.0
73.71	.558	178.8	.967	-149.1	.395	-133.0	.001	-116.9
78.50	.436	161.4	.959	-158.3	.514	-139.4	.001	-117.0
90.00	0.000	-180.0	0.000	-180.0	.894	-156.6	.072	-124.4

TABLE OF XMN COEF. IS FORMED

KXMAX=KYMAX= 1.38696 XY SPACING= .36050 WAVELENGTHS
KXM= 1.38696 KYM= .08669

SUBROUTINE NORM: MIN= 0. MAX= 0.

SUBROUTINE NORM: MIN= 0. MAX= .103E+01

```
IPOWER OF PATTERN= 1

 SUBROUTINE NORM:  MIN=  0.          MAX=  0.




 SUBROUTINE NORM:  MIN=  0.          MAX=   .103E+01




 SUBROUTINE NORM:  MIN=  0.          MAX=  0.




 SUBROUTINE NORM:  MIN=  0.          MAX=   .103E+01


TANGENT OGIVE PARAMETERS:  ROS(IN)=150.46975 BOS(IN)=142.33625
                          FINOS=3.000 FINIS= 3.07245
1      RESULTS OF RADOME ANALYSIS
TEST DATA TO TEST IBMRACP WITH PLOTS (CASE 3,F3,LINEAR,N=5)
FINENESS RATIO=  3.00  DIAMETER=16.26700 IN.  LENGTH=48.80200 IN.
FREQUENCY= 11.803 GHZ
PA= 7.74703 IN.  RR=39.76878 IN.  ANTENNA D=  5.1992 WAVELENGTHS
IPOL= 1  ICASE= 3  IOPT= 1

LAYER  THICKNESS(IN.)   ER      TAND

   1       .01525     6.000    .0090
   2       .17180     2.400    .0050
   3       .01525     6.000    .0090
   4       .17180     2.400    .0050
   5       .01525     6.000    .0090
```

```
 PHI   THETA   BSEEL     BSEAZ    SLPEL       SLPAZ       GAIN
(DEG)  (DEG)   (MRAD)    (MRAD)  (DEG/DEG)  (DEG/DEG)     (DB)


RECEIVING PATTERN COMPUTED FOR:
  ICUT=  1
 ICOMP=  1
  KMAX=     .996
  NREC=     32
    DK=    .62250E-01
ANGMAX=  84.87
(ICUT=1 FOR EL CUT, =2 FOR AZ CUT
(ICOMP=1 FOR EL COMPONENT, =2 FOR AZIMUTH)


RECEIVING PATTERN COMPUTED FOR:
  ICUT=  1
 ICOMP=  1
  KMAX=     .996
  NREC=     32
    DK=    .62250E-01
ANGMAX=  84.87
(ICUT=1 FOR EL CUT, =2 FOR AZ CUT
(ICOMP=1 FOR EL COMPONENT, =2 FOR AZIMUTH)


MIN AND MAX VALUES OF REC'G PATTERN:


 SUBROUTINE NORM:  MIN=    .101E-04  MAX=    .228E+03
```

RECVG PATTERN, EL CUT, EL COMP (DB)

-76.6	-40.000	78.7
-70.3	-40.000	77.8
-65.6	-40.000	77.2
-61.6	-40.000	66.3
-58.0	-40.000	-99.0
-54.8	-40.000	-103.2
-51.8	-40.000	-106.0
-49.0	-40.000	-107.6
-46.4	-40.000	-108.7
-43.9	-40.000	-166.1
-41.4	-40.000	71.6
-39.1	-37.260	68.2
-36.8	-35.283	65.4
-34.6	-35.754	63.1
-32.5	-40.000	60.2
-30.4	-40.000	-100.3
-28.3	-35.580	-118.9
-26.3	-30.221	-123.2
-24.4	-28.026	-127.5
-22.4	-28.432	-133.1
-20.5	-33.677	-149.2
-18.6	-33.095	78.1
-16.7	-21.696	56.4
-14.9	-15.563	49.6
-13.0	-11.306	44.8
-11.2	-8.095	40.7
-9.4	-5.591	37.0
-7.6	-3.625	33.5
-5.8	-2.113	30.6
-4.0	-1.014	28.3
-2.2	-.314	26.7
-.4	-.013	26.0
1.3	-.112	26.3
3.1	-.614	27.4
4.9	-1.514	29.3

6.7	-2.816	32.0
8.5	-4.546	35.2
10.3	-6.766	38.8
12.1	-9.599	42.7
14.0	-13.273	47.1
15.8	-18.294	52.5
17.7	-26.211	62.5
19.5	-40.000	157.6
21.5	-30.057	-138.2
23.4	-27.878	-129.7
25.3	-28.820	-125.2
27.3	-32.344	-121.5
29.4	-40.000	-116.2
31.4	-40.000	53.5
33.5	-37.178	61.7
35.7	-35.213	64.8
37.9	-35.968	67.0
40.2	-39.335	68.9
42.6	-40.000	72.9
45.1	-40.000	-113.2
47.7	-40.000	-108.6
50.4	-40.000	-106.0
53.3	-40.000	-104.3
56.4	-40.000	-102.9
59.7	-40.000	-95.8
63.5	-40.000	72.8
67.8	-40.000	77.0
73.2	-40.000	79.7
81.2	-40.000	80.6

MIN AND MAX VALUES OF REC"G PATTERN:

SUBROUTINE NORM: MIN= .644E-26 MAX= .102E+03

REC'G PATTERN, EL CUT, EL COMP (DB):

-76.6	-21.003	-9.8
-70.3	-21.534	-11.2
-65.6	-22.362	-14.4
-61.6	-20.830	-14.3
-58.0	-18.981	-13.5
-54.8	-18.031	-15.3
-51.8	-17.095	-17.7
-49.0	-15.637	-18.3
-46.4	-14.298	-18.5
-43.8	-13.500	-20.3
-41.4	-12.975	-22.6
-39.1	-12.458	-23.7
-36.8	-12.052	-24.4
-34.6	-11.815	-26.3
-32.5	-11.494	-28.5
-30.4	-10.840	-29.7
-28.3	-9.849	-30.6
-26.3	-8.590	-32.5
-24.4	-7.110	-34.7
-22.4	-5.534	-36.5
-20.5	-4.020	-38.4
-18.6	-2.668	-41.0
-16.7	-1.542	-44.0
-14.9	-.699	-46.8
-13.0	-.179	-49.7
-11.2	0.000	-52.9
-9.4	-.222	-56.1
-7.6	-.967	-58.9
-5.8	-2.418	-61.3
-4.0	-4.918	-63.4
-2.2	-9.523	-64.9
-.4	-23.261	-65.7
1.3	-13.798	114.6
3.1	-6.822	115.8
4.9	-3.504	117.6
6.7	-1.592	119.8

```
   8.5         -.314    122.5
  10.3         -.252    125.5
  12.1         -.353    128.7
  14.0         -.402    131.8
  15.8        -1.068    134.7
  17.7        -2.066    137.5
  19.5        -3.344    140.2
  21.5        -4.774    142.6
  23.4        -6.279    144.5
  25.3        -7.848    146.5
  27.3        -9.329    148.4
  29.4       -10.422    149.8
  31.4       -11.094    151.0
  33.5       -11.627    152.6
  35.7       -12.087    154.5
  37.9       -12.308    155.9
  40.2       -12.496    157.0
  42.6       -13.120    158.6
  45.1       -14.167    160.4
  47.7       -15.081    161.6
  50.4       -15.851    162.2
  53.3       -17.315    163.6
  56.4       -19.516    165.5
  59.7       -20.441    166.1
  63.5       -19.637    166.2
  67.8       -23.764    167.4
  73.2       -30.741    170.9
  81.2       -26.071    -11.3

RECEIVING PATTERN COMPUTED FOR:
  ICUT=  2
 ICOMP=  1
  KMAX=    .996
  NREC=    32
    DK=   .62250E-01
ANGMAX=  84.87
(ICUT=1 FOR EL CUT, =2 FOR AZ CUT
(ICOMP=1 FOR EL COMPONENT, =2 FOR AZIMUTH)
```

```
RECEIVING PATTERN COMPUTED FOR:
  ICUT=  2
 ICOMP=  1
  KMAX=     .996
  NREC=     32
    DK=   .62250E-01
ANGMAX=  84.87
(ICUT=1 FOR EL CUT, =2 FOR AZ CUT
(ICOMP=1 FOR EL COMPONENT, =2 FOR AZIMUTH)



MIN AND MAX VALUES OF REC"G PATTERN:



 SUBROUTINE NORM:  MIN=    .333E-03  MAX=    .229E+03



REC"G PATTERN, AZ CUT,EL COMP (DB):

   -76.6      -40.000    127.4
   -70.3      -40.000     94.2
   -65.6      -40.000     56.9
   -61.6      -40.000     23.4
   -58.0      -40.000   -116.4
   -54.8      -39.928   -105.7
   -51.8      -40.000    -87.3
   -49.0      -40.000    -97.8
   -46.4      -40.000   -113.9
   -43.8      -40.000   -135.2
   -41.4      -39.043     67.2
   -39.1      -35.989     70.6
   -36.8      -36.603     73.5
```

-34.6	-35.967	70.0	343
-32.5	-36.664	67.1	344
-30.4	-40.000	78.0	345
-28.3	-34.943	-116.5	346
-26.3	-30.795	-118.5	347
-24.4	-30.255	-122.5	348
-22.4	-30.934	-123.4	349
-20.5	-38.339	-114.9	350
-18.6	-29.339	50.2	351
-16.7	-19.348	51.3	352
-14.9	-14.037	48.1	353
-13.0	-10.218	44.8	354
-11.2	-7.115	42.8	355
-9.4	-4.693	40.8	356
-7.6	-2.928	37.9	357
-5.8	-1.640	34.7	358
-4.0	-.704	32.2	359
-2.2	-.145	29.9	360
-.4	-.002	26.9	361
1.3	-.234	23.4	362
3.1	-.815	20.9	363
4.9	-1.797	19.5	364
6.7	-3.237	17.0	365
8.5	-5.219	13.3	366
10.3	-8.013	13.9	367
12.1	-11.552	27.5	368
14.0	-14.019	48.4	369
15.8	-16.014	48.5	370
17.7	-18.720	25.1	371
19.5	-23.462	-.4	372
21.5	-39.094	-110.6	373
23.4	-26.638	-170.7	374
25.3	-28.957	-161.4	375
27.3	-36.279	-119.9	376
29.4	-40.000	-119.0	377
31.4	-40.000	134.8	378
33.5	-38.563	48.8	379
35.7	-32.843	25.7	380

37.9	-34.848	36.1
40.2	-40.000	78.6
42.6	-40.000	82.8
45.1	-40.000	-33.7
47.7	-40.000	-110.6
50.4	-37.358	-138.4
53.3	-39.200	-126.8
56.4	-40.000	-78.0
59.7	-40.000	-73.0
63.5	-40.000	163.6
67.8	-40.000	89.6
73.2	-40.000	51.7
81.2	-40.000	64.1

MIN AND MAX VALUES OF REC"G PATTERN:

SUBROUTINE NORM: MIN= .634E-02 MAX= .987E+02

REC"G PATTERN, AZ CUT,EL COMP (DB):

-76.6	-20.835	-9.9
-70.3	-21.287	-9.5
-65.6	-22.002	-11.6
-61.6	-20.511	-11.0
-58.0	-18.741	-9.7
-54.8	-17.779	-11.2
-51.8	-16.811	-13.7
-49.0	-15.418	-14.0
-46.4	-14.172	-13.9
-43.8	-13.400	-15.6
-41.4	-12.873	-18.0
-39.1	-12.419	-19.0
-36.8	-12.098	-19.6
-34.6	-11.970	-21.5

-32.5	-11.512	-24.1	419
-30.4	-10.874	-25.5	420
-28.3	-9.907	-26.8	421
-26.3	-8.589	-28.9	422
-24.4	-7.027	-31.3	423
-22.4	-5.426	-33.1	424
-20.5	-3.906	-35.0	425
-18.6	-2.520	-37.3	426
-16.7	-1.369	-39.7	427
-14.9	-.555	-42.0	428
-13.0	-.104	-44.3	429
-11.2	-.014	-46.8	430
-9.4	-.369	-49.4	431
-7.6	-1.328	-52.0	432
-5.8	-3.071	-54.7	433
-4.0	-5.955	-57.2	434
-2.2	-11.276	-58.9	435
-.4	-33.185	-42.0	436
1.3	-12.689	112.4	437
3.1	-6.709	109.9	438
4.9	-3.650	107.0	439
6.7	-1.879	104.9	440
8.5	-.957	103.9	441
10.3	-.579	103.4	442
12.1	-.443	103.7	443
14.0	-.460	105.7	444
15.8	-.818	109.4	445
17.7	-1.724	112.5	446
19.5	-3.141	112.9	447
21.5	-4.750	111.1	448
23.4	-6.359	110.9	449
25.3	-8.050	114.5	450
27.3	-9.664	119.0	451
29.4	-10.813	121.2	452
31.4	-11.471	122.7	453
33.5	-12.034	126.3	454
35.7	-12.523	130.6	455
37.9	-12.673	133.1	456

```
    40.2       -12.745     135.1
    42.6       -13.325     138.6
    45.1       -14.361     142.6
    47.7       -15.188     145.1
    50.4       -15.839     146.9
    53.3       -17.275     150.2
    56.4       -19.510     154.5
    59.7       -20.371     156.7
    63.5       -19.466     158.0
    67.8       -20.596     161.3
    73.2       -30.702     163.1
    81.2       -25.865     -14.0
NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        169
NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        169
NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        169
NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        169
NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        169


FINAL ANSWERS FOR MONOPULSE SYSTEM:

K1:  -.53886E-02 -.35755E-04   .99999E+00
K2:  -.53667E-02  .14448E-07   .99999E+00

AZTM=   -.53667E+01 MRAD
ELTM=    .14449E-04 MRAD

MESAZ=    .93465E-01 VOLTS/DEG
MESEL=    .10196E+00 VOLTS/DEG

UAZ:  -.11735E-03  .45944E-07
UEL:  -.20898E-03  .84448E-07

SMAX=    .12096280585064E+03     LCTR=    3
```

```
ADDITIONAL MONOPULSE OUTPUTS AROUND BORESIGHT:

NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        169
ANG=  -3.0 MRAD FROM BORESIGHT    VRAZ=  -.16052E-01 VOLTS    VREL=  -.17502E-0

DAZ(AMP,PHS)=   .16703E-01 -.73959E+02   DEL(AMP,PHS)=   .17507E-01 -.91343E+02

NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        169
ANG=  -2.0 MRAD FROM BORESIGHT    VRAZ=  -.10706E-01 VOLTS    VREL=  -.11675E-0

DAZ(AMP,PHS)=   .11671E-01 -.65530E+02   DEL(AMP,PHS)=   .11678E-01 -.91340E+02

NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        169
ANG=  -1.0 MRAD FROM BORESIGHT    VRAZ=  -.53557E-02 VOLTS    VREL=  -.58410E-0

DAZ(AMP,PHS)=   .71136E-02 -.48840E+02   DEL(AMP,PHS)=   .58426E-02 -.91340E+02

NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        169
ANG=   0.0 MRAD FROM BORESIGHT    VRAZ=   .45944E-07 VOLTS    VREL=   .84448E-0

DAZ(AMP,PHS)=   .47179E-02  .55796E-03   DEL(AMP,PHS)=   .84471E-07  .88662E+02

NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        169
ANG=   1.0 MRAD FROM BORESIGHT    VRAZ=   .53599E-02 VOLTS    VREL=   .58486E-0

DAZ(AMP,PHS)=   .71646E-02  .48426E+02   DEL(AMP,PHS)=   .58502E-02  .88664E+02

NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        169
ANG=   2.0 MRAD FROM BORESIGHT    VRAZ=   .10724E-01 VOLTS    VREL=   .11705E-0

DAZ(AMP,PHS)=   .11746E-01  .65927E+02   DEL(AMP,PHS)=   .11708E-01  .88665E+02

NUMBER OF RAYS USED IN COMPUTING APERTURE FIELD =        169
ANG=   3.0 MRAD FROM BORESIGHT    VRAZ=   .16093E-01 VOLTS    VREL=   .17570E-0

DAZ(AMP,PHS)=   .16832E-01  .73301E+02   DEL(AMP,PHS)=   .17575E-01  .88667E+02
```

```
AVERAGE SLPAZ=    .93500E-01 VOLTS/DEG
AVERAGE SLPEL=    .10201E+00 VOLTS/DEG
 SUM=1.0 VOLT

  0.0  14.0      .00    -5.37    1.0000    0.0000    -1.6

RECEIVED SUM VOLTAGE WITHOUT RADOME=  .14615E+03
```

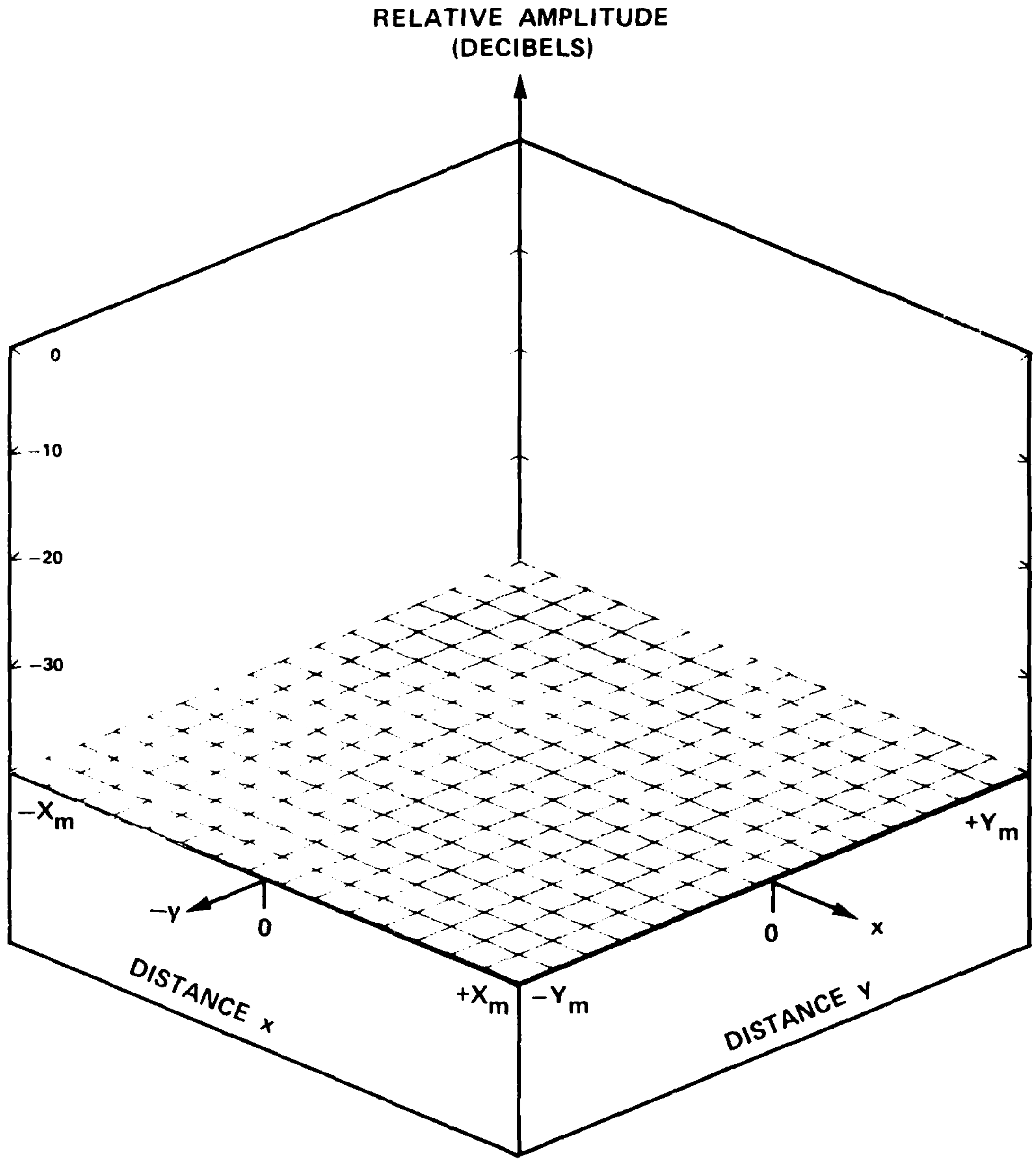

Figure D-1. $|E_x|$ of Flat Plate Antenna (ICASE=3) for Sum, Elevation Difference, and Azimuth Difference Channels.

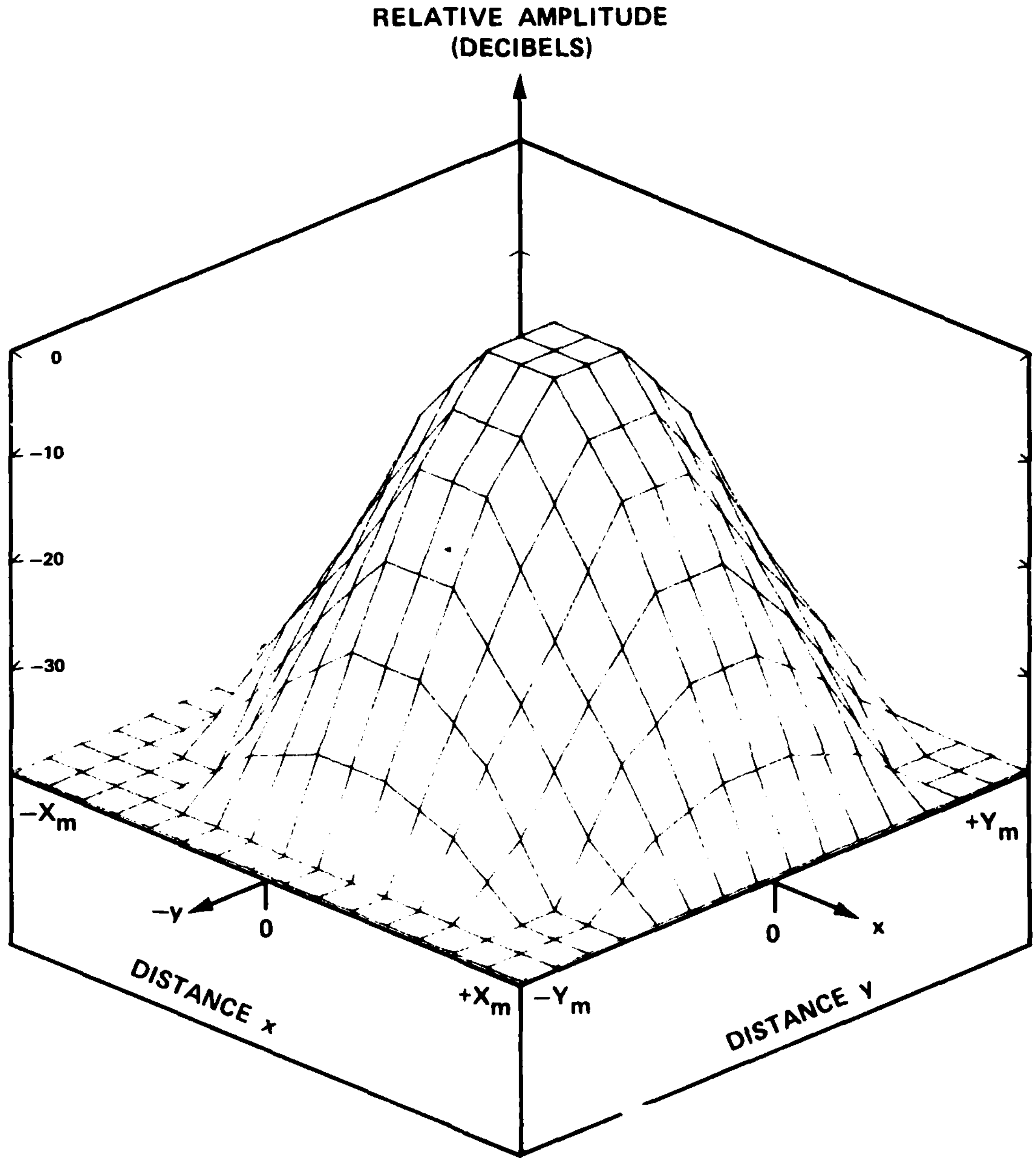

Figure D-2. $|E_Y|_\Sigma$ of Flat Plate Antenna.

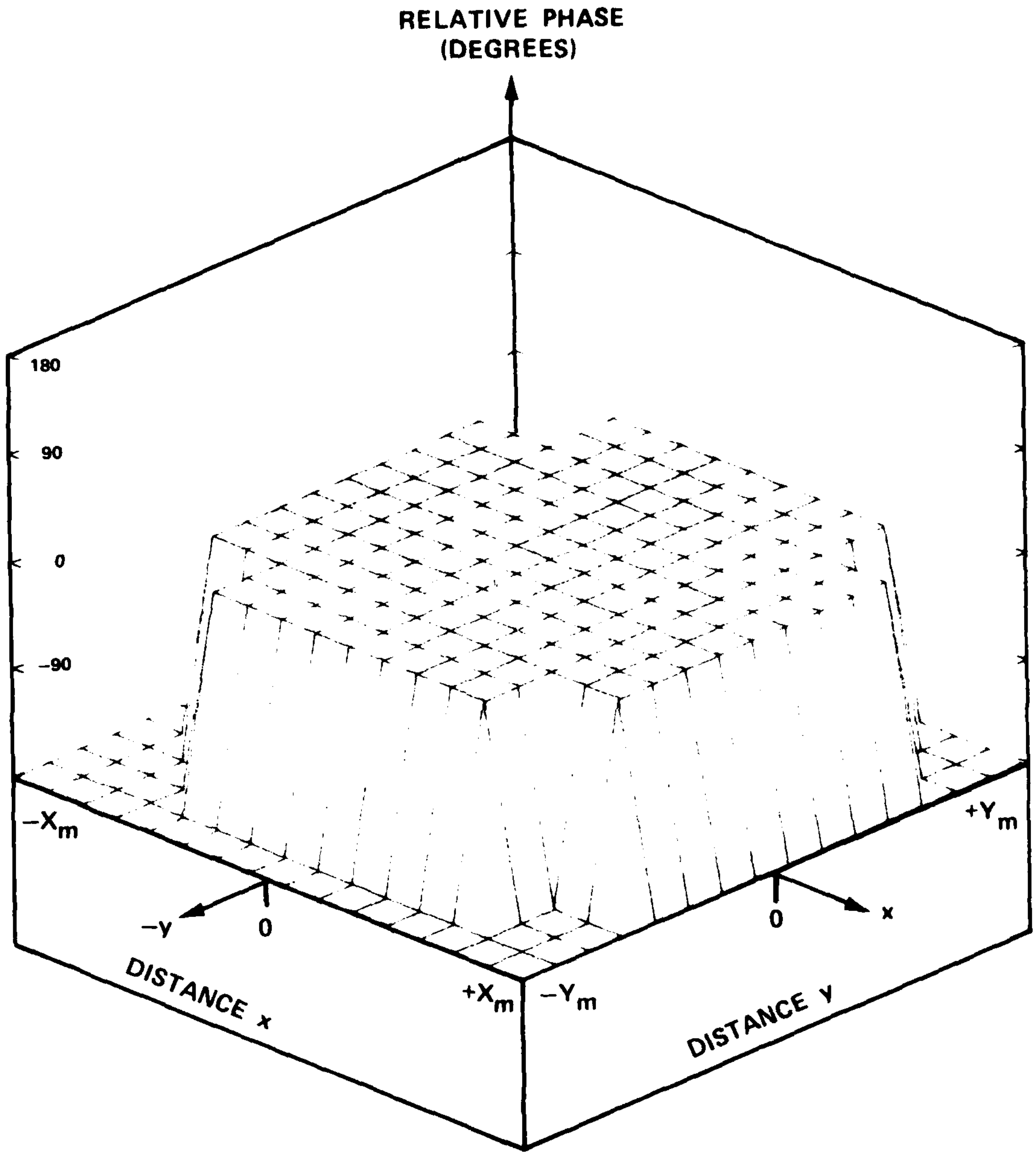

Figure D-3. Phase of $E_{Y\Sigma}$ of Flat Plate Antenna.

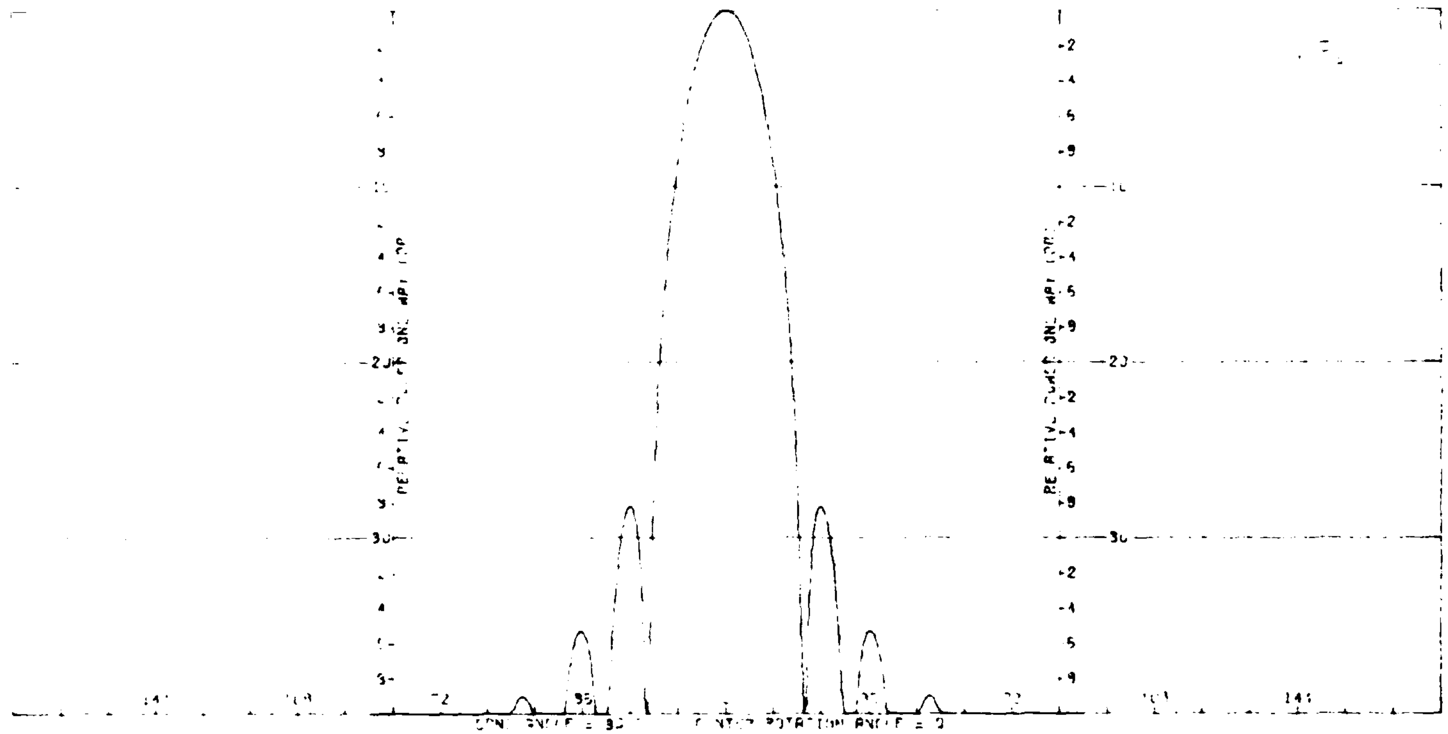

Figure D-4. Transmitting E-Plane Sum Pattern of Flat Plate Antenna.

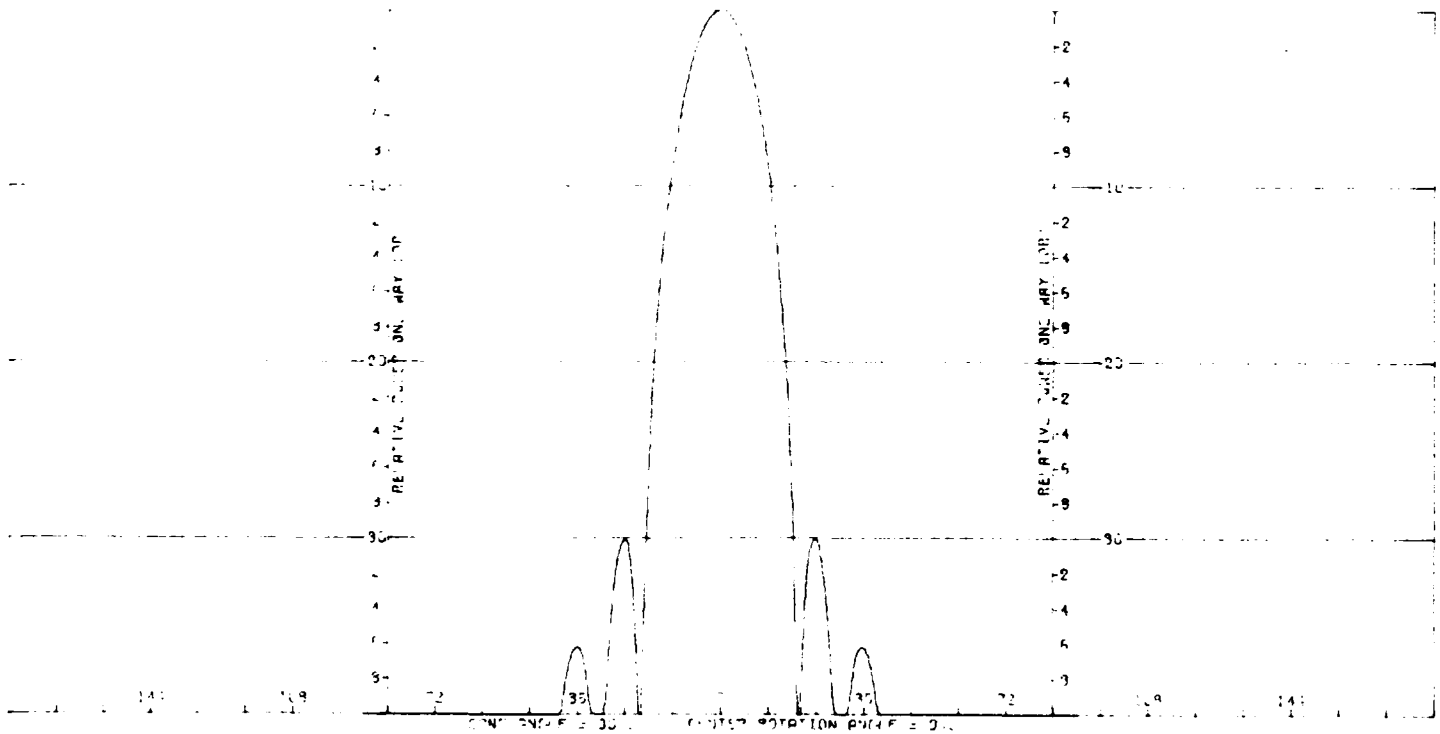

Figure D-5. Transmitting H-Plane Sum Pattern of Flat Plate Antenna.

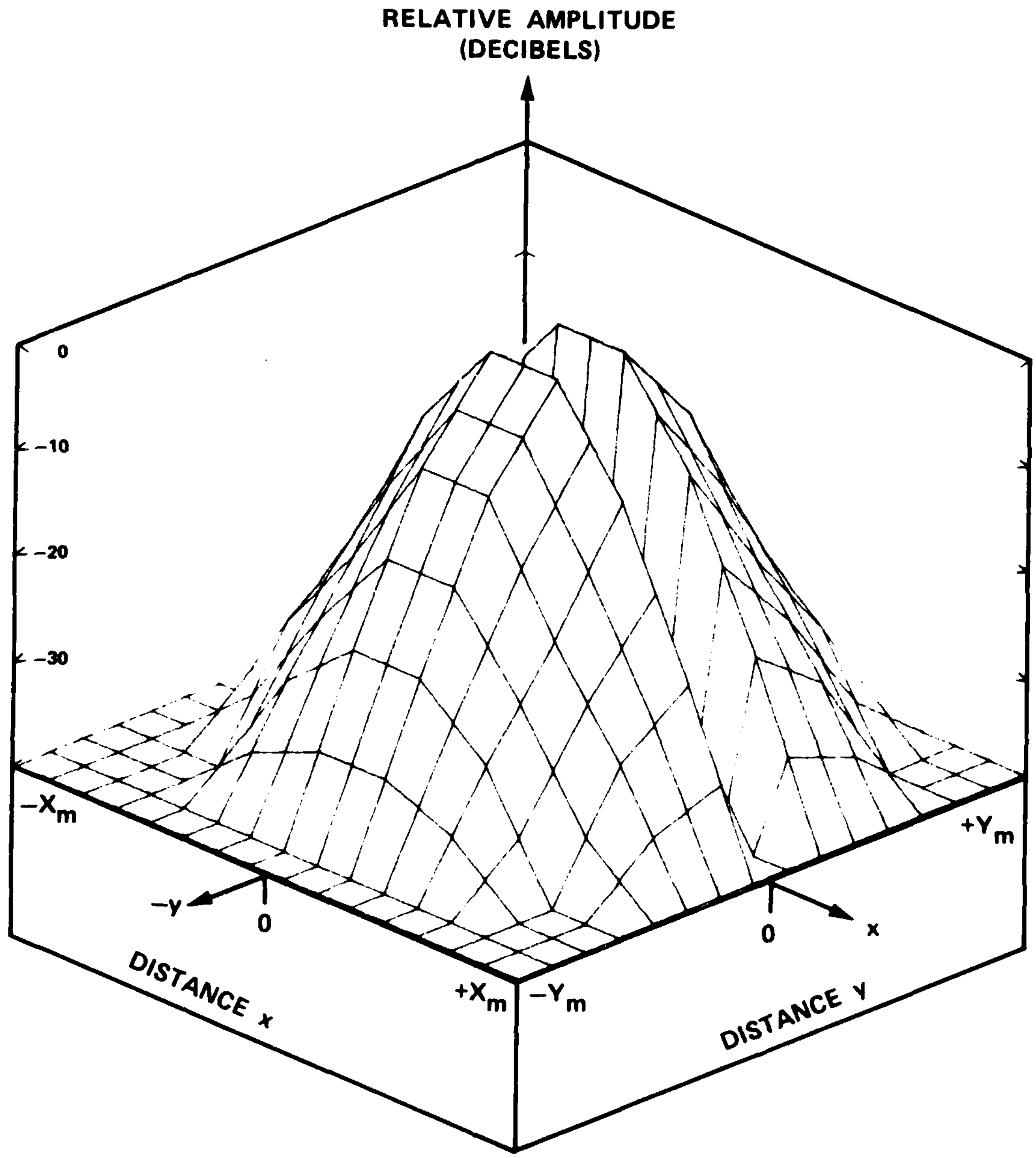

Figure D-6. $|E_{Y\Delta EL}|$ of Flat Plate Antenna.

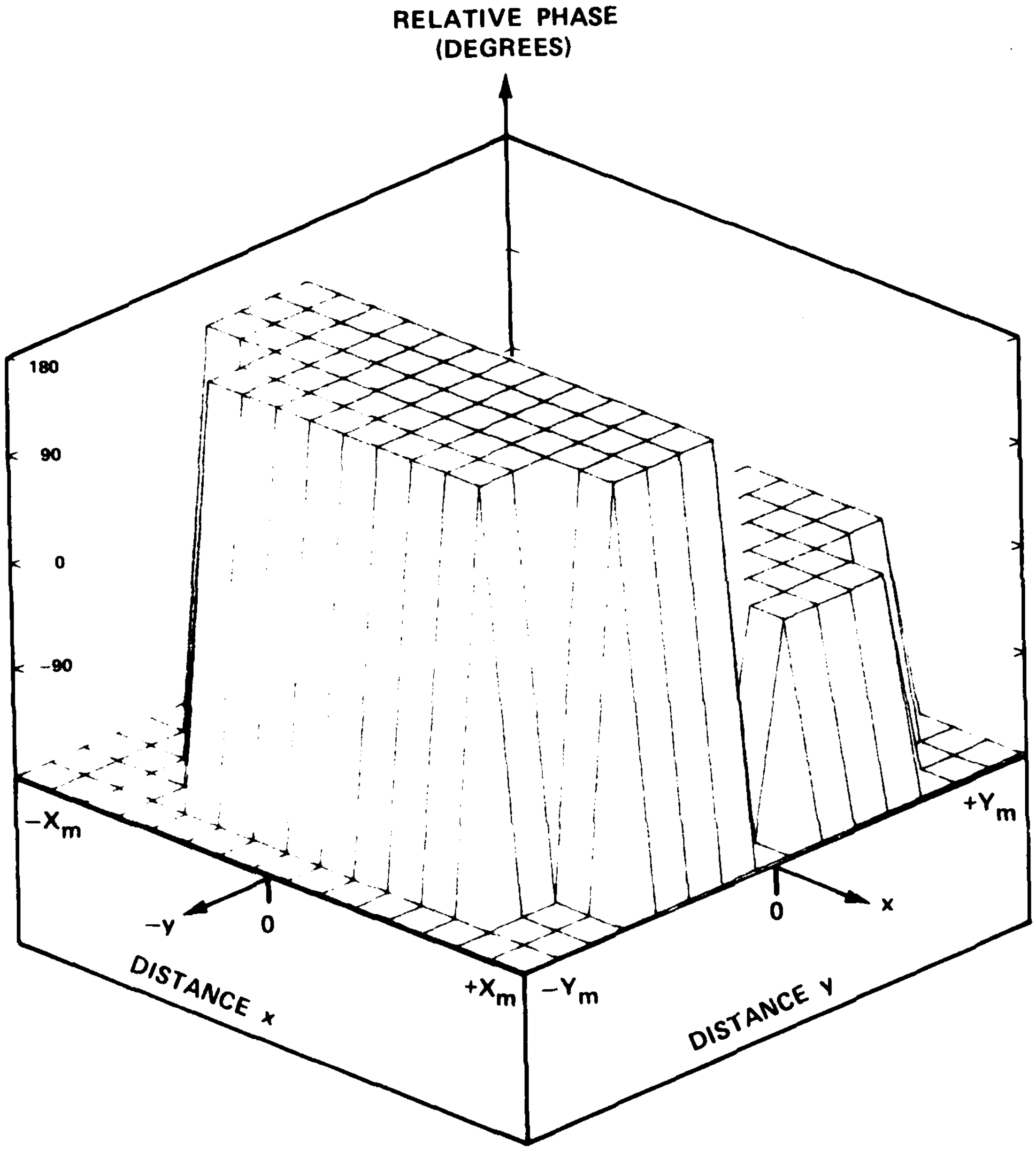

Figure D-7. Phase of $E_{Y\Delta EL}$ of Flat Plate Antenna.

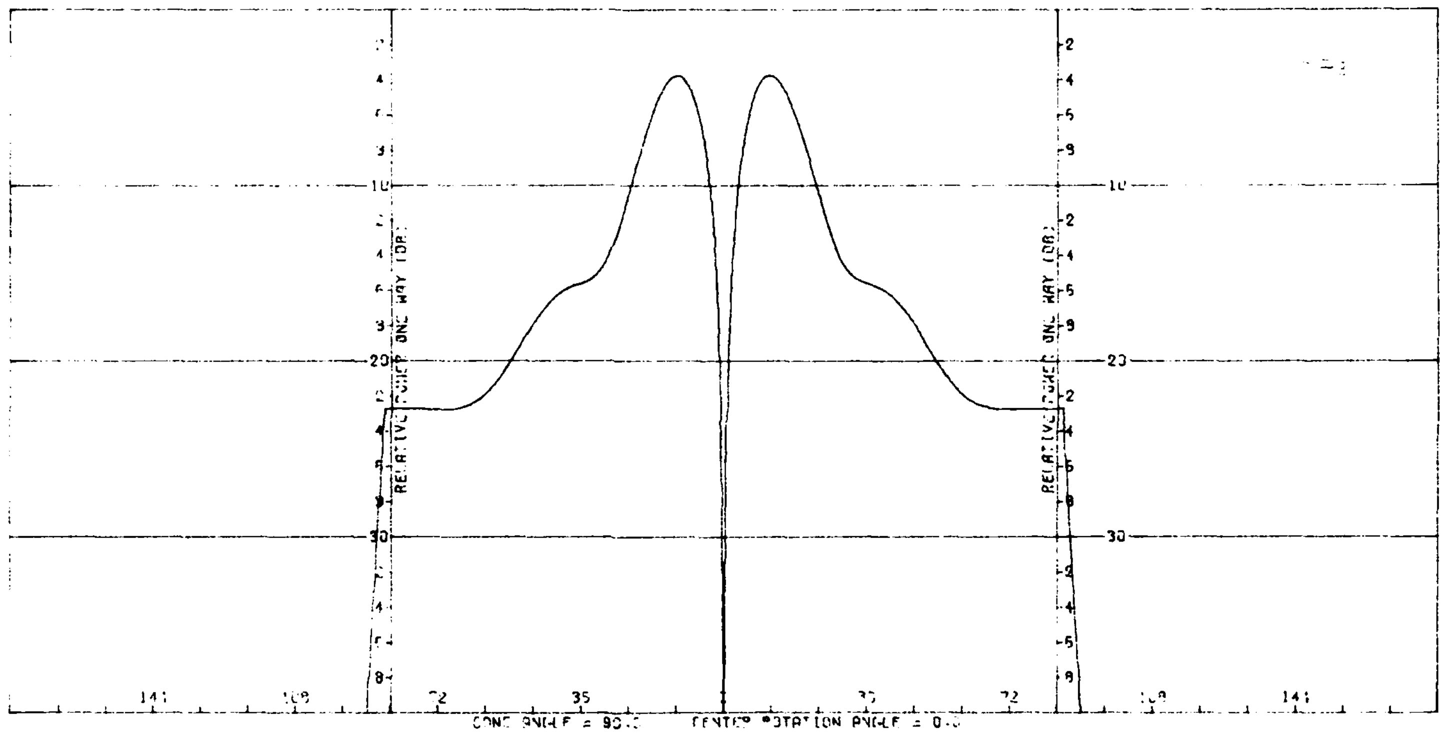

Figure D-8. Transmitting E-Plane Δ_{EL} Pattern of Flat Plate Antenna.

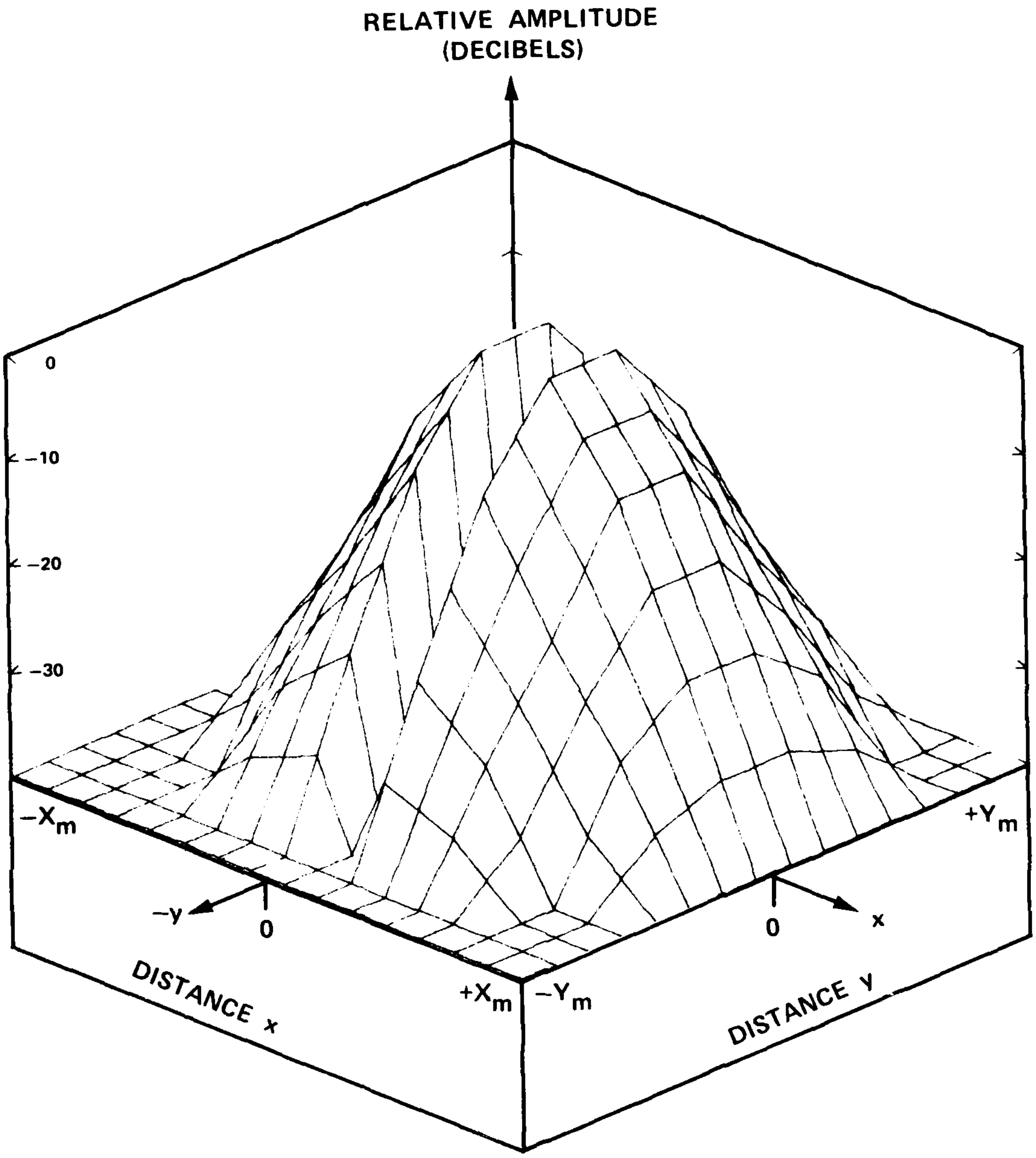

Figure D-9. $|E_{Y\Delta AZ}|$ of Flat Plate Antenna.

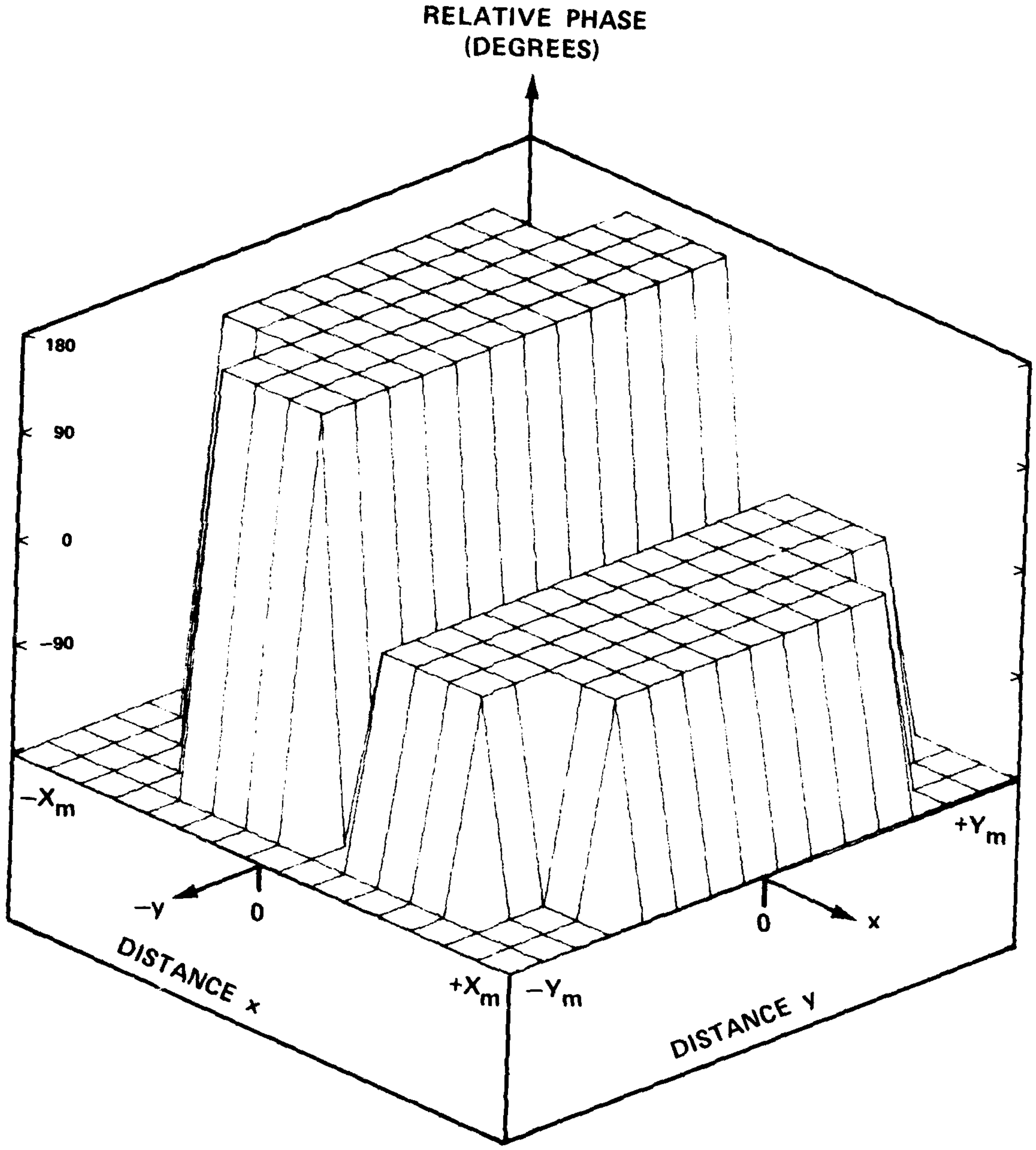

Figure D-10. Phase of $E_{Y\Delta AZ}$ of Flat Plate Antenna.

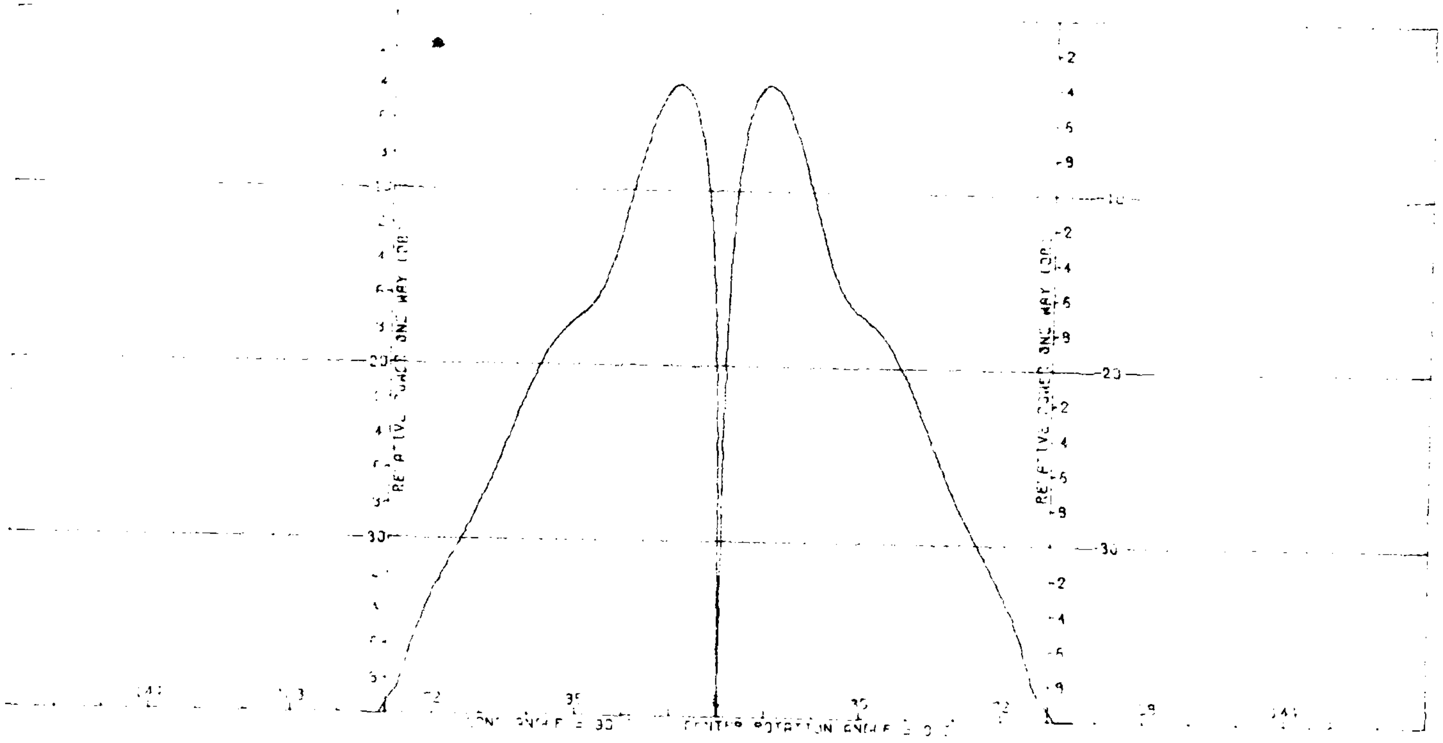

Figure D-11. Transmitting H-Plane Δ_{AZ} Pattern of Flat Plate Antenna.

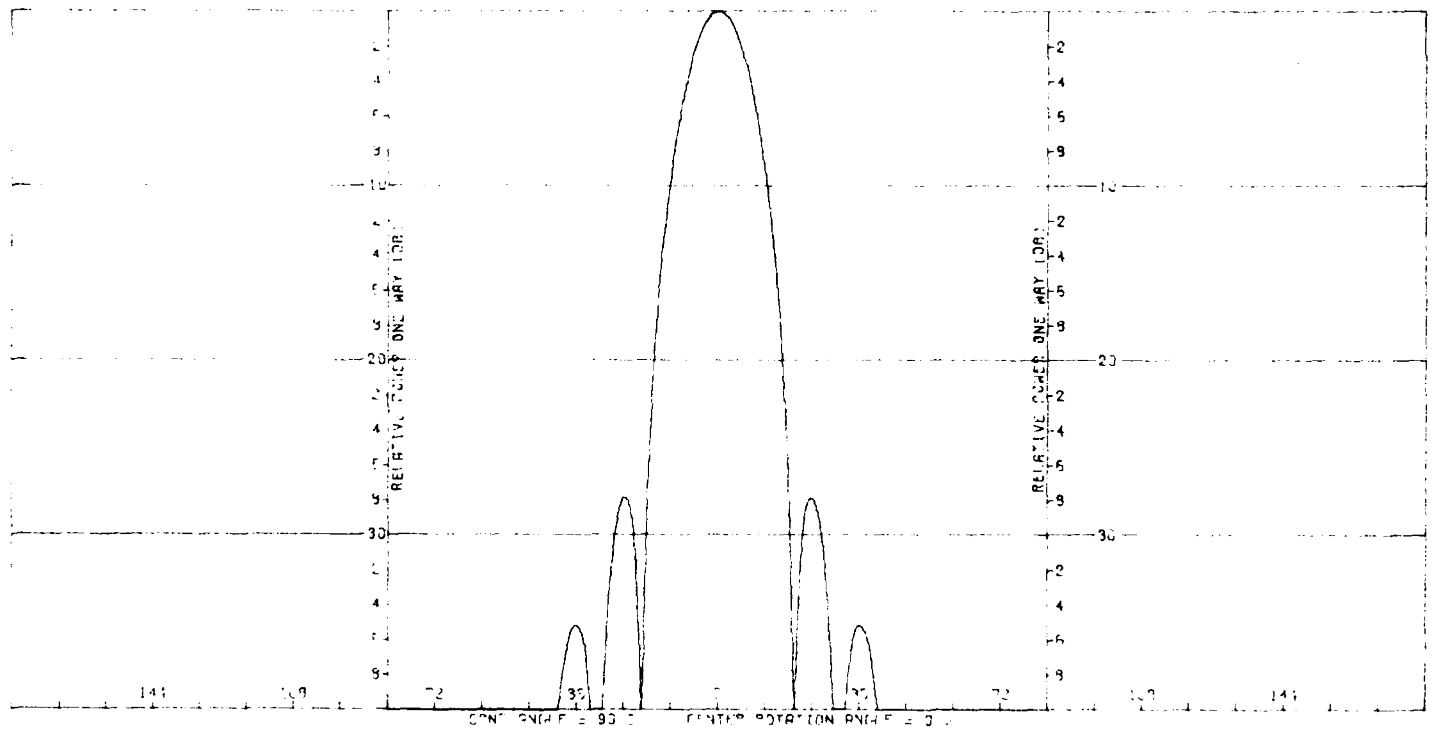

Figure D-12. Receiving E-Plane Sum Pattern of Flat Plate Antenna With Radome.

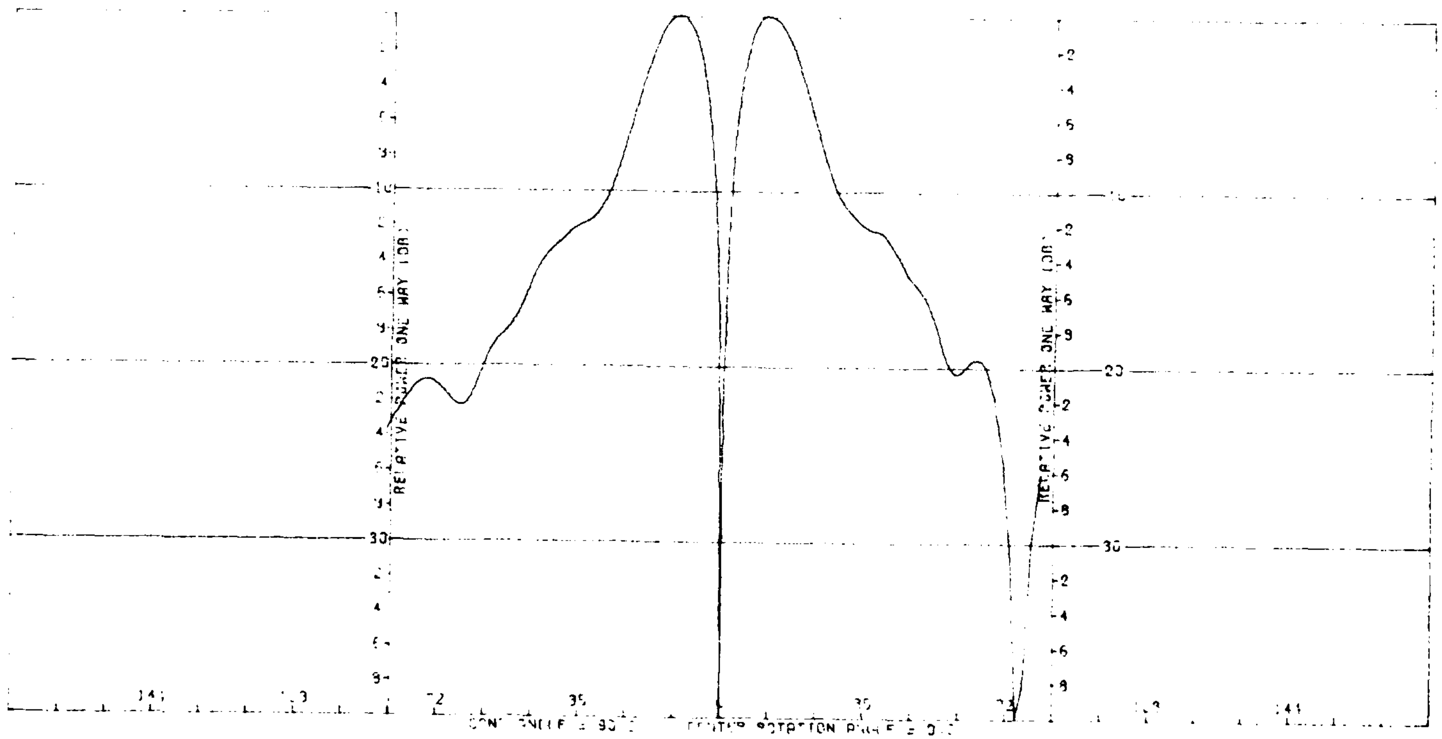

Figure D-13. Receiving E-Plane Λ_{EL} Pattern of Flat Plate Antenna With Radome.

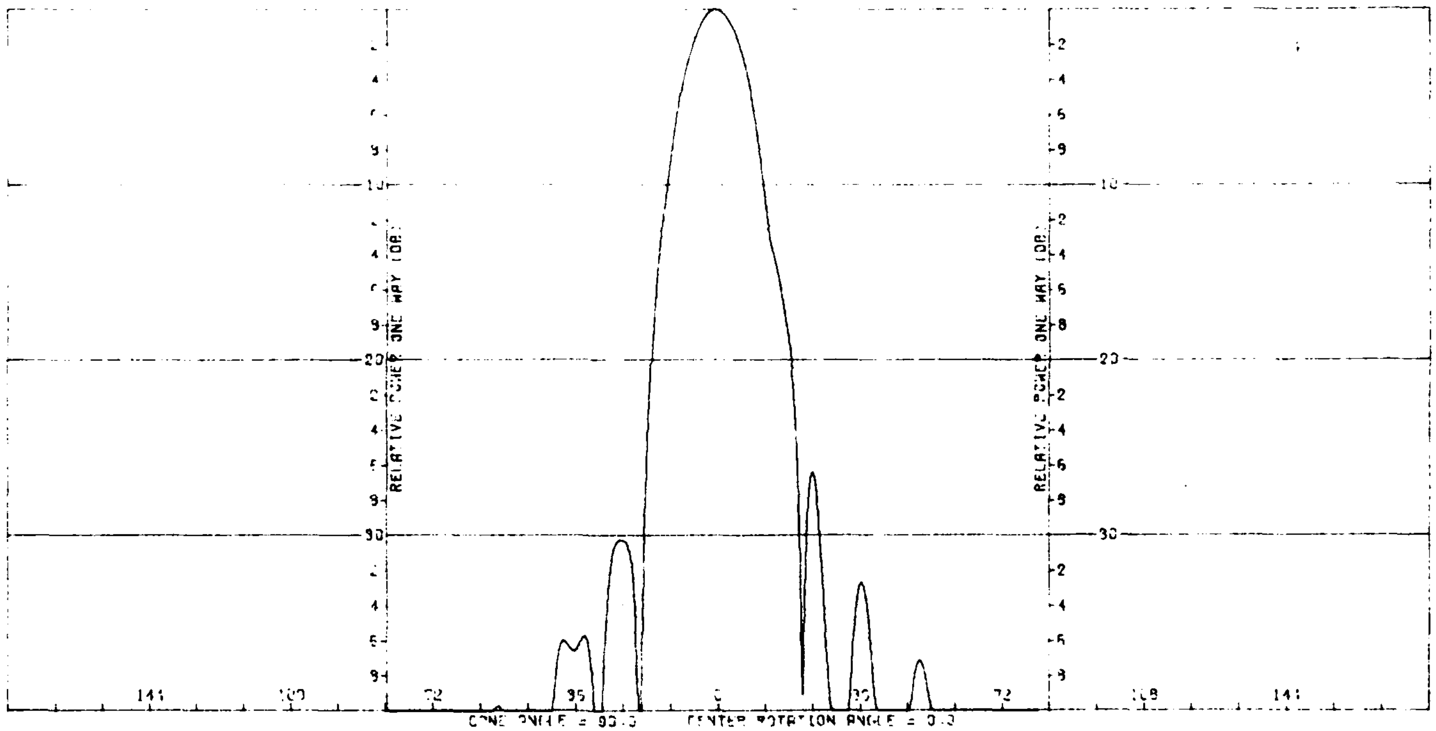

Figure D-14. Receiving H-Plane Sum Pattern of Flat Plate Antenna With Radome.

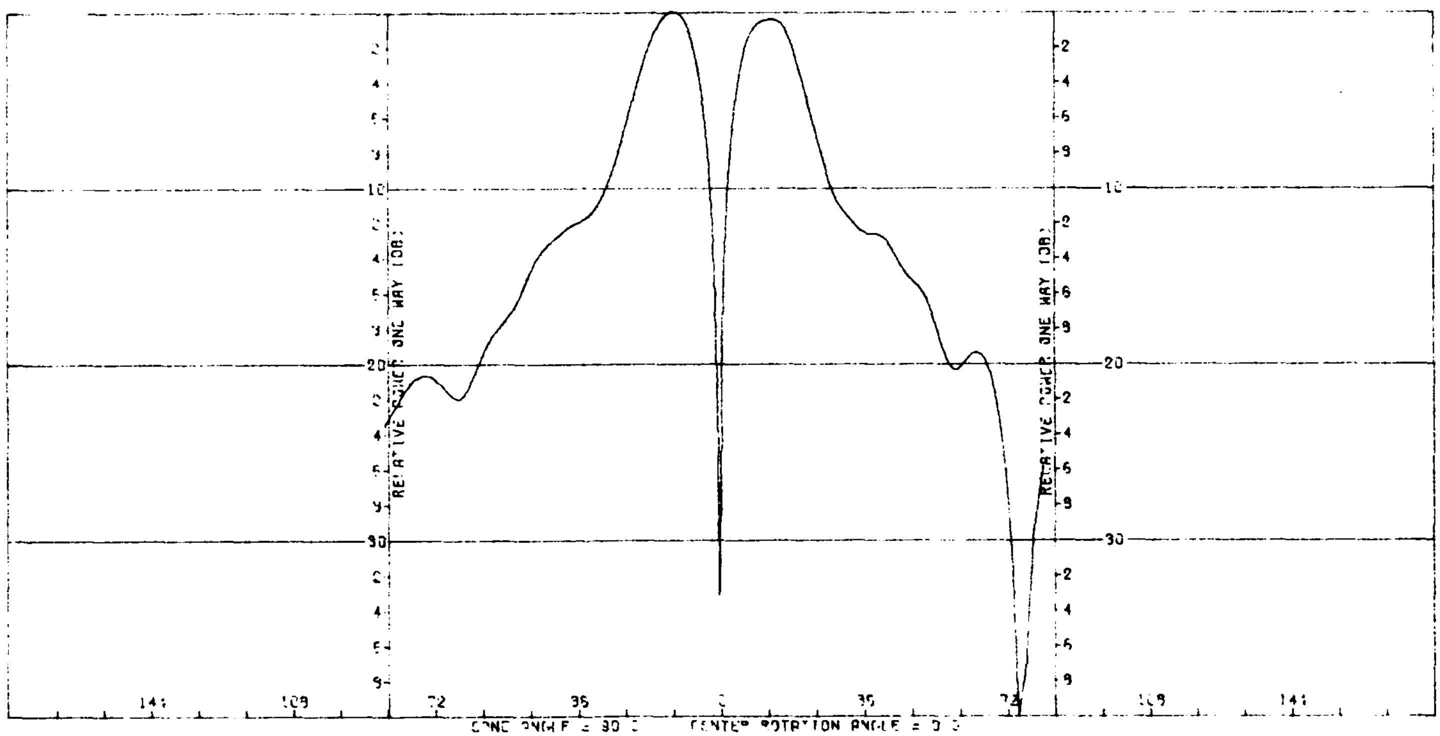

Figure D-15. Receiving H-Plane Δ_{AZ} Pattern of Flat Plate Antenna With Radome.

Appendix E

Plane Wave Transmission Through Multilayered Radome Wall

(Excerpted from Reference 1 cited in Chapter 11.)

The derivation below and the computer program implementation listed in Appendix F are based on work done by Richmond at Ohio State University. Although Richmond's matrix formulation for the analysis of plane multilayers has been previously documented [3], an outline of the theory is repeated here to provide a convenient reference in defining the quantities described in the computer program of Appendix F.

Consider a plane electromagnetic wave incident on the surface of a stack of plane, homogeneous, dielectric slabs of finite thickness and infinite width surrounded by free space as shown in Figure 7(a). The wave illustrated has perpendicular polarization (electric field intensity vector perpendicular to the plane of incidence) and the symbols $\underline{E}_i$ and $\underline{E}_r$ represent the electric field intensities of the incident and reflected waves at the "incident point " P, and $\underline{E}_t$ represents the electric field intensity of the transmitted wave at the "normal exit point " Q. The reflection coefficient R and the "normal transmission coefficient " T_n of the multilayer are defined by

$$R = \frac{E_r(P)}{E_i(P)} \quad \text{(perpendicular polarization)} \tag{171}$$

and

$$T_n = \frac{E_t(Q)}{E_i(P)} \quad \text{(perpendicular polarization)} \tag{172}$$

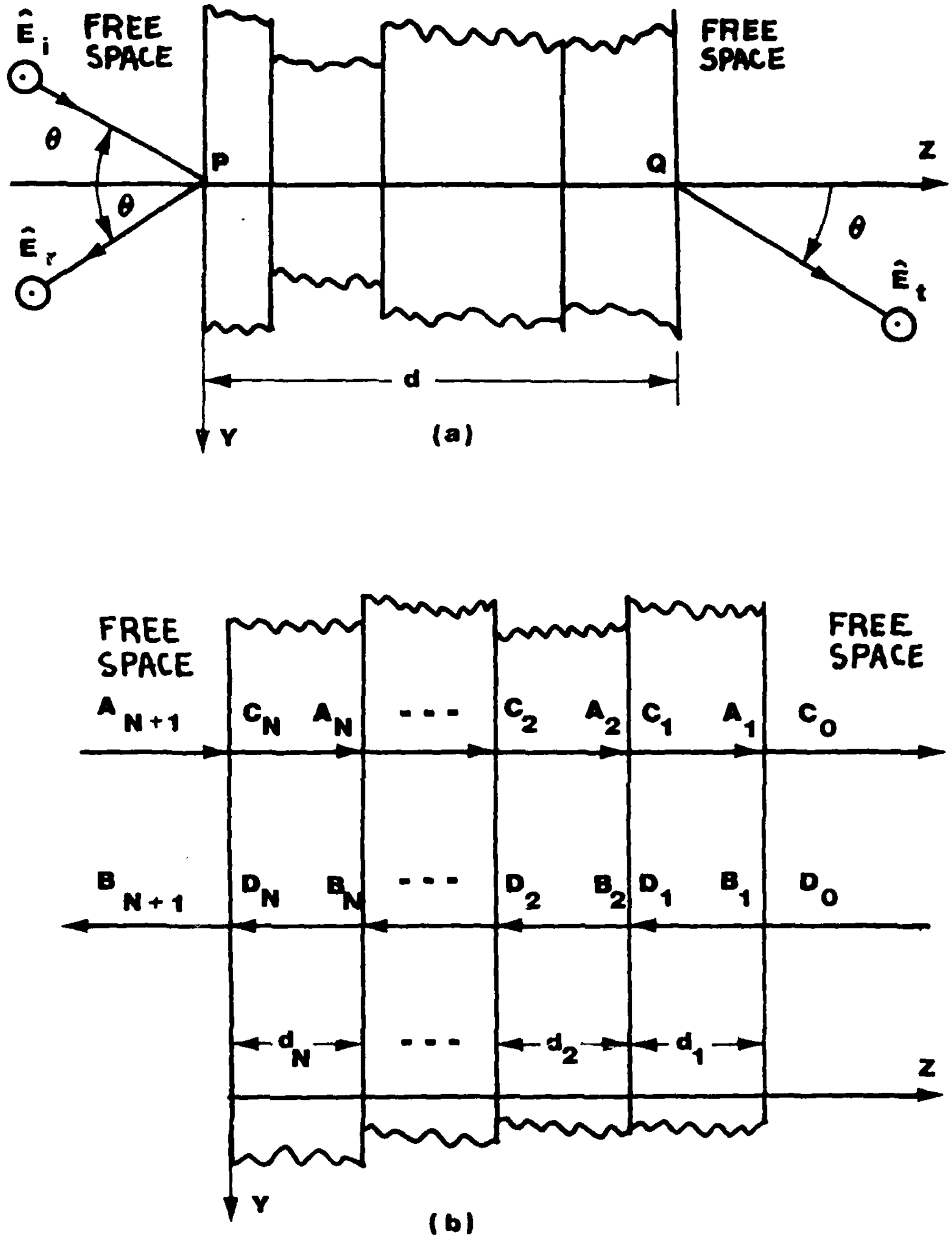

Figure 7. Plane Electromagnetic Wave Incident on Plane Multilayer.

The "insertion transmission coefficient" T is defined as follows

$$T = \frac{E_t(Q)}{E_i(Q)} \text{ (perpendicular polarization)} \tag{172}$$

$$= T_n e^{jkd\cos\theta}$$

where d is the total multilayer thickness, θ is the angle of incidence measured from the normal, and k is the free-space phase constant $\omega\sqrt{\mu_o \varepsilon_o} = 2\pi/\lambda_o$.

The resultant field in each layer consists of an outgoing wave and a reflected wave. In Figure 1 the complex constants A_n and C_n represent the electric field intensity E_x of the outgoing wave in layer n, evaluated at its two boundaries, and B_n and D_n represent the reflected field intensity at the two boundaries.

The field intensity in layer n can be written as

$$E_x = (ae^{-\gamma_n z} + be^{\gamma_n z})e^{-jky\sin\theta} \tag{173}$$

The propagation constant γ_n is expressed in terms of the attenuation constant α_n and the phase constant β_n as

$$\gamma_n = \alpha_n + j\beta_n \tag{174}$$

It is assumed that the permeability of each layer is real and the complex permittivity is expressed as

$$\dot{\epsilon} = \epsilon'(1-j\tan\delta) \tag{175}$$

Using the wave equations and Equations (173), (174), and (175), it can be found that

$$\alpha = (k/\sqrt{2}\,)\sqrt{\sqrt{(\mu_r\epsilon_r' - \sin^2\theta)^2 + (\mu_r\epsilon_r'\tan\delta)^2} - (\mu_r\epsilon_r' - \sin^2\theta)} \tag{176}$$

$$\beta = (k/\sqrt{2}\,)\sqrt{\sqrt{(\mu_r\epsilon_r' - \sin^2\theta)^2 + (\mu_r\epsilon_r'\tan\delta)^2} + (\mu_r\epsilon_r' - \sin^2\theta)} \tag{177}$$

where μ_r and ϵ_r' are the relative permeability and permittivity:

$$\mu_r = \mu/\mu_o \tag{178}$$

and

$$\epsilon_r' = \epsilon'/\epsilon_o \tag{179}$$

Evaluating E_x in Equation (173) at the left and right boundaries of layer n, it can be shown that

$$A_n = C_n e^{-\gamma_n d_n} \tag{180}$$

and

$$B_n = D_n e^{\gamma_n d_n} \tag{181}$$

where d_n is the thickness of layer n. Equations (180) and (181) can be expressed by the following matrix equation:

$$\begin{bmatrix} A_n \\ \\ B_n \end{bmatrix} = \begin{bmatrix} e^{-\gamma_n d_n} & 0 \\ \\ 0 & e^{\gamma_n d_n} \end{bmatrix} \begin{bmatrix} C_n \\ \\ D_n \end{bmatrix} . \tag{182}$$

Let $t_{n+1,n}$ and $r_{n+1,n}$ denote the interface transmission and reflection coefficients for a wave in layer n+1 incident on the boundary of layer n. Further, let $t_{n,n+1}$ and $r_{n,n+1}$ represent the interface coefficients for a wave in layer n on the boundary of layer n+1. In terms of these coefficients, the electric field intensities, evaluated at both sides of the boundary between layers n and n+1, are related linearly as follows:

$$C_n = t_{n+1,n}A_{n+1} + r_{n,n+1}D_n \tag{183}$$

and

$$B_{n+1} = t_{n,n+1}D_n + r_{n+1,n}A_{n+1} \tag{184}$$

The relations follow from the superposition theorem and the definitions of the interface coefficients.

It can be shown that

$$r_{n,n+1} = -r_{n+1,n} \tag{185}$$

$$t_{n+1,n} = 1 + r_{n+1,n} \tag{186}$$

$$t_{n,n+1} = 1 + r_{n,n+1} = 1 - r_{n+1,n}, \tag{187}$$

and

$$t_{n+1,n}t_{n,n+1} - r_{n+1,n}r_{n,n+1} = 1 \tag{188}$$

By using Equations (185) through (188), Equations (183) and (184) can be arranged as

$$C_n = (A_{n+1} + r_{n,n+1}B_{n+1})/t_{n,n+1} \tag{189}$$

and

$$D_n = (B_{n+1} - r_{n+1,n}A_{n+1})/t_{n,n+1} \tag{190}$$

These can be expressed in matrix form as

$$\begin{pmatrix} C_n \\ \\ D_n \end{pmatrix} = \frac{1}{t_{n,n+1}} \begin{pmatrix} 1 & -r_{n+1,n} \\ \\ -r_{n+1,n} & 1 \end{pmatrix} \begin{pmatrix} A_{n+1} \\ \\ B_{n+1} \end{pmatrix} . \tag{191}$$

The matrix Equations (182) and (191) can be combined to obtain the following:

$$\begin{pmatrix} C_{n-1} \\ \\ D_{n-1} \end{pmatrix} = \frac{1}{t_{n-1,n}} \begin{pmatrix} e^{-\gamma_n d_n} & -r_{n,n-1}e^{\gamma_n d_n} \\ \\ -r_{n,n-1}e^{-\gamma_n d_n} & e^{\gamma_n d_n} \end{pmatrix} \begin{pmatrix} C_n \\ \\ D_n \end{pmatrix} \tag{192}$$

Let the two-by-two matrix in Equation (192) be denoted by M_n:

$$M_n = \begin{bmatrix} e^{-\gamma_n d_n} & -r_{n,n-1} e^{\gamma_n d_n} \\ -r_{n,n-1} e^{-\gamma_n d_n} & e^{\gamma_n d_n} \end{bmatrix} \tag{193}$$

Repeated application of Equation (193) yields the following matrix relationship between the electric field intensities at the incidence and exit surfaces:

$$\begin{bmatrix} C_o \\ D_o \end{bmatrix} = (1/t)\ M_1 \cdot M_2 \cdot M_3 \ldots M_N \cdot S \cdot \begin{bmatrix} A_{N+1} \\ B_{N+1} \end{bmatrix} \tag{194}$$

where the dots denote matrix multiplication, N represents the total number of layers, S denotes the matrix

$$S = \begin{bmatrix} 1 & -r_{N+1,N} \\ -r_{N+1,N} & 1 \end{bmatrix}, \tag{195}$$

and

$$t = t_{0,1} \cdot t_{1,2} \cdot t_{2,3} \cdots t_{N,N+1}. \tag{196}$$

In the situation used to define the transmission and reflection coefficients of the structure, a wave of unit amplitude is assumed to

be incident on one outer surface, so that

$$A_{N+1} = 1 \tag{197}$$

$$B_{N+1} = R \tag{198}$$

$$C_o = T_n \tag{199}$$

and

$$D_o = 0 \tag{200}$$

Thus Equation (194) becomes

$$\begin{bmatrix} T_n \\ 0 \end{bmatrix} = (1/t)\, M_1 \cdot M_2 \cdot M_3 \ldots M_N \cdot S \cdot \begin{bmatrix} 1 \\ R \end{bmatrix} \tag{201}$$

The solution for "parallel polarization" (electric field intensity parallel to the plane of incidence) is obtained by applying the theorem of duality to the above solution. Thus, the reflection and transmission coefficients are defined by

[illegible] = $\frac{H_{z}(I)}{H_{z}(I)}$ (parallel polarization) [illegible]

and

[illegible] = $\frac{H_{z}([illegible])}{H_{z}(P)}$ (parallel polarization) [illegible]

The matrix equations given above apply also for parallel polarization, in which case the complex constants A_n, B_n, C_n and D_n represent the amplitudes of the *magnetic* field intensities H_x of the traveling waves in layer n. Equations (185) through (188) also apply for parallel polarization, in which case the interface reflection and transmission coefficients are defined by the ratio of the *magnetic* field intensities H_x. The interface reflection coefficients are given by

$$r_{n+1,n} = \frac{\mu_n \gamma_{n+1} - \mu_{n+1}\gamma_n}{\mu_n \gamma_{n+1} + \mu_{n+1}\gamma_n} \quad \text{(perpendicular polarization)} \tag{204}$$

and

$$r_{n+1,n} = \frac{\dot{\varepsilon}_n \gamma_{n+1} - \dot{\varepsilon}_{n+1}\gamma_n}{\dot{\varepsilon}_n \gamma_{n+1} + \dot{\varepsilon}_{n+1}\gamma_n} \quad \text{(parallel polarization)} \tag{205}$$

where γ is given by Equations (174), (176), and (177) if the permeability μ of each layer is real.

After the indicated matrix multiplications of Equation (44) are performed, and the division by t, the equation has the form

$$\begin{pmatrix} T_n \\ 0 \end{pmatrix} \quad \begin{pmatrix} a & b \\ c & e \end{pmatrix}\begin{pmatrix} 1 \\ R \end{pmatrix} \tag{206}$$

Thus,

$$T_n = a + bR = a - \frac{bc}{e} \tag{207}$$

and

$$R = -c/e. \tag{208}$$

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